

11 Integraalrekening

Voorkennis Differentiaalrekening

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- 1** a $f(x) = 4 \ln^2(x) - 3 \ln(x)$ geeft $f'(x) = 8 \ln(x) \cdot \frac{1}{x} - 3 \cdot \frac{1}{x} = \frac{8 \ln(x) - 3}{x}$
- b $f(x) = \frac{10 \cdot 3^x}{\ln(3)} = \frac{10}{\ln(3)} \cdot 3^x$ geeft $f'(x) = \frac{10}{\ln(3)} \cdot 3^x \cdot \ln(3) = 10 \cdot 3^x$
- c $f(x) = x^2 \cdot \sqrt{2x-1}$ geeft
 $f'(x) = 2x \cdot \sqrt{2x-1} + x^2 \cdot \frac{1}{2\sqrt{2x-1}} \cdot 2 = 2x\sqrt{2x-1} + \frac{x^2}{\sqrt{2x-1}} = \frac{2x(2x-1) + x^2}{\sqrt{2x-1}} = \frac{5x^2 - 2x}{\sqrt{2x-1}}$
- d $f(x) = \frac{e^{2x} + 3}{4e^x} = \frac{1}{4}e^x + \frac{3}{4}e^{-x}$ geeft $f'(x) = \frac{1}{4}e^x + \frac{3}{4}e^{-x} \cdot -1 = \frac{e^x}{4} - \frac{3}{4e^x} = \frac{e^{2x} - 3}{4e^x}$
- 2** a $f(x) = x \ln(x) - x$ geeft $f'(x) = 1 \cdot \ln(x) + x \cdot \frac{1}{x} - 1 = \ln(x) + 1 - 1 = \ln(x)$
- b $f(x) = x\sqrt{4x-3}$ geeft
 $f'(x) = 1 \cdot \sqrt{4x-3} + x \cdot \frac{1}{2\sqrt{4x-3}} \cdot 4 = \sqrt{4x-3} + \frac{2x}{\sqrt{4x-3}} = \frac{4x-3+2x}{\sqrt{4x-3}} = \frac{6x-3}{\sqrt{4x-3}}$
- c $f(x) = \frac{e^{4x} - 5}{2e^{3x}} = \frac{1}{2}e^x - \frac{5}{2}e^{-3x}$ geeft $f'(x) = \frac{1}{2}e^x - \frac{5}{2}e^{-3x} \cdot -3 = \frac{1}{2}e^x + \frac{15}{2e^{3x}} = \frac{e^{4x} + 15}{2e^{3x}}$
- d $f(x) = x^2 \ln^2(x) - x^2 \ln(x)$ geeft
 $f'(x) = 2x \cdot \ln^2(x) + x^2 \cdot 2 \ln(x) \cdot \frac{1}{x} - 2x \cdot \ln(x) - x^2 \cdot \frac{1}{x} = 2x \ln^2(x) + 2x \ln(x) - 2x \ln(x) - x = 2x \ln^2(x) - x$
- e $f(x) = 6x^2\sqrt{x^2+1}$ geeft
 $f'(x) = 12x \cdot \sqrt{x^2+1} + 6x^2 \cdot \frac{1}{2\sqrt{x^2+1}} \cdot 2x = 12x\sqrt{x^2+1} + \frac{6x^3}{\sqrt{x^2+1}} = \frac{12x(x^2+1) + 6x^3}{\sqrt{x^2+1}} = \frac{18x^3 + 12x}{\sqrt{x^2+1}}$
- f $f(x) = \frac{5 \cdot 2^x}{\ln(2)} - \frac{e^{2x} - 1}{e^x} = \frac{5}{\ln(2)} \cdot 2^x - (e^x - e^{-x}) = \frac{5}{\ln(2)} \cdot 2^x - e^x + e^{-x}$ geeft
 $f'(x) = \frac{5}{\ln(2)} \cdot 2^x \cdot \ln(2) - e^x + e^{-x} \cdot -1 = 5 \cdot 2^x - \left(e^x + \frac{1}{e^x}\right) = 5 \cdot 2^x - \frac{e^{2x} + 1}{e^x}$
- 3** a $f(x) = x + \frac{1}{1+e^x} - \ln(1+e^x) = x + (1+e^x)^{-1} - \ln(1+e^x)$ geeft
 $f'(x) = 1 - (1+e^x)^{-2} \cdot e^x - \frac{1}{1+e^x} \cdot e^x = 1 - \frac{e^x}{(1+e^x)^2} - \frac{e^x}{1+e^x}$
 $= \frac{(1+e^x)^2 - e^x - e^x(1+e^x)}{(1+e^x)^2} = \frac{1 + 2e^x + e^{2x} - e^x - e^x - e^{2x}}{(1+e^x)^2} = \frac{1}{(1+e^x)^2}$
- b $f(x) = \sin(x) - \frac{1}{3}\sin^3(x)$ geeft
 $f'(x) = \cos(x) - \sin^2(x) \cdot \cos(x) = \cos(x) - (1 - \cos^2(x)) \cdot \cos(x) = \cos(x) - \cos(x) + \cos^3(x) = \cos^3(x)$
- c $f(x) = x + 2 - \frac{4}{x+2} - 4 \ln(x+2) = x + 2 - 4(x+2)^{-1} - 4 \ln(x+2)$ geeft
 $f'(x) = 1 + 4(x+2)^{-2} - 4 \cdot \frac{1}{x+2} = 1 + \frac{4}{(x+2)^2} - \frac{4}{x+2}$
 $= \frac{(x+2)^2 + 4 - 4(x+2)}{(x+2)^2} = \frac{x^2 + 4x + 4 + 4 - 4x - 8}{(x+2)^2} = \frac{x^2}{(x+2)^2}$

d $f(x) = (0,2x^2 - 0,4x + 1,2)\sqrt{2x+3}$ geeft

$$\begin{aligned} f'(x) &= (0,4x - 0,4) \cdot \sqrt{2x+3} + (0,2x^2 - 0,4x + 1,2) \cdot \frac{1}{2\sqrt{2x+3}} \cdot 2 \\ &= (0,4x - 0,4)\sqrt{2x+3} + \frac{0,2x^2 - 0,4x + 1,2}{\sqrt{2x+3}} = \frac{(0,4x - 0,4)(2x+3)}{\sqrt{2x+3}} + \frac{0,2x^2 - 0,4x + 1,2}{\sqrt{2x+3}} \\ &= \frac{0,8x^2 + 1,2x - 0,8x - 1,2 + 0,2x^2 - 0,4x + 1,2}{\sqrt{2x+3}} = \frac{x^2}{\sqrt{2x+3}} \end{aligned}$$

11.1 Primitieven en integralen

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- 1 $g(x) = x^3 + c$ geeft $g'(x) = 3x^2$ en dit is $f(x)$.
 $g(2) = 15$ geeft $2^3 + c = 15$ oftewel $8 + c = 15$, dus $c = 7$.

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- 2 a $F(x) = \frac{1}{9}(3x+4)^3$ geeft $F'(x) = \frac{1}{3}(3x+4)^2 \cdot 3 = (3x+4)^2$
Dus $F'(x) = f(x)$ oftewel F is een primitieve van f .
b $F(x) = x - \ln(x+2)$ geeft $F'(x) = 1 - \frac{1}{x+2} = \frac{x+2-1}{x+2} = \frac{x+1}{x+2}$
Dus $F'(x) = f(x)$ oftewel F is een primitieve van f .
c $F(x) = \frac{-1}{2(x^2+1)} = -\frac{1}{2}(x^2+1)^{-1}$ geeft $F'(x) = \frac{1}{2}(x^2+1)^{-2} \cdot 2x = \frac{x}{(x^2+1)^2}$
Dus $F'(x) = f(x)$ oftewel F is een primitieve van f .
3 a $F(x) = (x^2+1)^6 + 1$ geeft $F'(x) = 6(x^2+1)^5 \cdot 2x = 12x(x^2+1)^5$
Dus $F'(x) = f(x)$ oftewel F is een primitieve van f .
b $G(x) = (\frac{1}{2}x - \frac{1}{4})e^{2x} - 2$ geeft $G'(x) = \frac{1}{2} \cdot e^{2x} + (\frac{1}{2}x - \frac{1}{4}) \cdot e^{2x} \cdot 2 = (\frac{1}{2} + x - \frac{1}{2}) \cdot e^{2x} = x e^{2x}$
Dus $G'(x) = g(x)$ oftewel G is een primitieve van g .
c $H(x) = \ln^2(x) + 2 \ln(x) + 3$ geeft $H'(x) = 2 \ln(x) \cdot \frac{1}{x} + 2 \cdot \frac{1}{x} = \frac{2 \ln(x) + 2}{x}$
Dus $H'(x) = h(x)$ oftewel H is een primitieve van h .
d $J(x) = \frac{e^{3x} - 10}{2e^x} - 4 = \frac{1}{2}e^{2x} - 5e^{-x} - 4$ geeft $J'(x) = \frac{1}{2}e^{2x} \cdot 2 - 5e^{-x} \cdot -1 = e^{2x} + \frac{5}{e^x} = \frac{e^{3x} + 5}{e^x}$
Dus $J'(x) = j(x)$ oftewel J is een primitieve van j .
e $K(x) = \sin^3(x)$ geeft $K'(x) = 3 \sin^2(x) \cdot \cos(x) = 3(1 - \cos^2(x)) \cdot \cos(x) = 3 \cos(x) - 3 \cos^3(x)$
Dus $K'(x) = k(x)$ oftewel K is een primitieve van k .

4 $F(x) = \ln(x + \sqrt{x^2 - 1})$ geeft

$$\begin{aligned} F'(x) &= \frac{1}{x + \sqrt{x^2 - 1}} \cdot \left(1 + \frac{1}{2\sqrt{x^2 - 1}} \cdot 2x\right) = \frac{1}{x + \sqrt{x^2 - 1}} \cdot \left(1 + \frac{x}{\sqrt{x^2 - 1}}\right) = \frac{1}{x + \sqrt{x^2 - 1}} \cdot \frac{\sqrt{x^2 - 1} + x}{\sqrt{x^2 - 1}} \\ &= \frac{\sqrt{x^2 - 1} + x}{x + \sqrt{x^2 - 1}} \cdot \frac{1}{\sqrt{x^2 - 1}} = \frac{1}{\sqrt{x^2 - 1}} \end{aligned}$$

Dus $F'(x) = f(x)$ oftewel F is een primitieve van f .

- 5 a $F(x) = \frac{1}{5}x^5$ geeft $F'(x) = x^4$
Dus $F(x) = \frac{1}{5}x^5$ is een primitieve van $f(x) = x^4$.
b $G(x) = \frac{1}{4}e^{4x+1}$ geeft $G'(x) = \frac{1}{4}e^{4x+1} \cdot 4 = e^{4x+1}$
Dus $G(x) = \frac{1}{4}e^{4x+1}$ is een primitieve van $g(x) = e^{4x+1}$.
c $H(x) = \frac{3^x}{\ln(3)} = \frac{1}{\ln(3)} \cdot 3^x$ geeft $H'(x) = \frac{1}{\ln(3)} \cdot 3^x \cdot \ln(3) = 3^x$
Dus $H(x) = \frac{3^x}{\ln(3)}$ is een primitieve van $h(x) = 3^x$.
d $K(x) = \frac{1}{2} \sin(2x)$ geeft $K'(x) = \frac{1}{2} \cdot \cos(2x) \cdot 2 = \cos(2x)$
Dus $K(x) = \frac{1}{2} \sin(2x)$ is een primitieve van $k(x) = \cos(2x)$.

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6 $F(x) = a \cdot \frac{1}{n+1} x^{n+1} + c$ geeft $F'(x) = a \cdot \frac{1}{n+1} \cdot (n+1)x^n = ax^n$

Dus $F(x) = a \cdot \frac{1}{n+1} x^{n+1} + c$ zijn de primitieven van $f(x) = ax^n$.

$$F(x) = \frac{g^x}{\ln(g)} + c = \frac{1}{\ln(g)} \cdot g^x + c \text{ geeft } F'(x) = \frac{1}{\ln(g)} \cdot g^x \cdot \ln(g) = g^x$$

Dus $F(x) = \frac{g^x}{\ln(g)} + c$ zijn de primitieven van $f(x) = g^x$.

$$F(x) = e^x + c \text{ geeft } F'(x) = e^x$$

Dus $F(x) = e^x + c$ zijn de primitieven van $f(x) = e^x$.

$$F(x) = \ln|x| + c = \begin{cases} \ln(x) + c & \text{voor } x > 0 \\ \ln(-x) + c & \text{voor } x < 0 \end{cases} \text{ geeft } F'(x) = \begin{cases} \frac{1}{x} & \text{voor } x > 0 \\ -\frac{1}{x} \cdot -1 = \frac{1}{x} & \text{voor } x < 0 \end{cases}$$

Dus $F(x) = \ln|x| + c$ zijn de primitieven van $f(x) = \frac{1}{x}$.

$$F(x) = x \ln(x) - x + c \text{ geeft } F'(x) = 1 \cdot \ln(x) + x \cdot \frac{1}{x} - 1 = \ln(x)$$

Dus $F(x) = x \ln(x) - x + c$ zijn de primitieven van $f(x) = \ln(x)$.

$$F(x) = \frac{1}{\ln(g)} (x \ln(x) - x) + c \text{ geeft}$$

$$F'(x) = \frac{1}{\ln(g)} \cdot \left(1 \cdot \ln(x) + x \cdot \frac{1}{x} - 1 \right) = \frac{1}{\ln(g)} \cdot \ln(x) = \frac{\ln(x)}{\ln(g)} = {}^g\log(x)$$

Dus $F(x) = \frac{1}{\ln(g)} (x \ln(x) - x) + c$ zijn de primitieven van $f(x) = {}^g\log(x)$.

$$F(x) = -\cos(x) + c \text{ geeft } F'(x) = \sin(x)$$

Dus $F(x) = -\cos(x) + c$ zijn de primitieven van $f(x) = \sin(x)$.

$$F(x) = \sin(x) + c \text{ geeft } F'(x) = \cos(x)$$

Dus $F(x) = \sin(x) + c$ zijn de primitieven van $f(x) = \cos(x)$.

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7 a $f(x) = ax^{-1}$ geeft $F(x) = a \cdot \frac{1}{-1+1} x^{-1+1} + c$ kan niet kloppen, want $\frac{1}{-1+1} = \frac{1}{0}$ bestaat niet en $x^0 = 1$ voor $x \neq 0$.

b $f(x) = x^{-1} = \frac{1}{x}$ geeft $F(x) = \ln|x| + c$

c $g(x) = a \cdot F(x)$ geeft $g'(x) = a \cdot F'(x) = a \cdot f(x)$

Dus als F een primitieve is van f , dan is $a \cdot F$ een primitieve van $a \cdot f$.

8 a $f(x) = 6x^2$ geeft $F(x) = 6 \cdot \frac{1}{3} x^3 + c = 2x^3 + c$

b $f(x) = 2x^3 + 5x^4$ geeft $F(x) = 2 \cdot \frac{1}{4} x^4 + 5 \cdot \frac{1}{5} x^5 + c = \frac{1}{2} x^4 + x^5 + c$

c $f(x) = \frac{x^4 - 2x}{2x^3} = \frac{1}{2} x - x^{-2}$ geeft $F(x) = \frac{1}{4} x^2 + x^{-1} + c = \frac{1}{4} x^2 + \frac{1}{x} + c$

d $f(x) = 10^x$ geeft $F(x) = \frac{10^x}{\ln(10)} + c$

e $f(x) = 5 \cdot 2^x$ geeft $F(x) = 5 \cdot \frac{2^x}{\ln(2)} + c = \frac{5 \cdot 2^x}{\ln(2)} + c$

f $f(x) = x^2 + \sin(x)$ geeft $F(x) = \frac{1}{3} x^3 - \cos(x) + c$

g $f(x) = \frac{x^3 + 2}{x^4} = \frac{1}{x} + 2x^{-4}$ geeft $F(x) = \ln|x| + 2 \cdot -\frac{1}{3} \cdot x^{-3} + c = \ln|x| - \frac{2}{3x^3} + c$

h $f(x) = x\sqrt{x} - 2\cos(x) = x^{1/2} - 2\cos(x)$ geeft

$$F(x) = \frac{1}{2\frac{1}{2}} x^{2\frac{1}{2}} - 2\sin(x) + c = \frac{2}{5} x^2 \cdot \sqrt{x} - 2\sin(x) + c$$

- 9** a $f(x) = x^3 - 3x$ geeft $F(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2 + c$
 b $f(x) = 5e^x$ geeft $F(x) = 5e^x + c$
 c $f(x) = \frac{x^4 - 6}{2x^3} = \frac{1}{2}x - 3x^{-3}$ geeft $F(x) = \frac{1}{4}x^2 + \frac{3}{2}x^{-2} + c = \frac{1}{4}x^2 + \frac{3}{2x^2} + c$
 d $f(x) = 3^x + x^3$ geeft $F(x) = \frac{3^x}{\ln(3)} + \frac{1}{4}x^4 + c$
 e $f(x) = 2\ln(x)$ geeft $F(x) = 2(x\ln(x) - x) + c = 2x\ln(x) - 2x + c$
 f $f(x) = \ln(2x) = \ln(2) + \ln(x)$ geeft $F(x) = \ln(2) \cdot x + x\ln(x) - x + c = x\ln(2) + x\ln(x) - x + c$

- 10** a $f(x) = e^{x+1} = e^x \cdot e = e \cdot e^x$ geeft $F(x) = e \cdot e^x + c = e^{x+1} + c$
 b $f(x) = \frac{8}{x^3} = 8x^{-3}$ geeft $F(x) = 8 \cdot -\frac{1}{2}x^{-2} + c = -\frac{4}{x^2} + c$
 c $f(x) = \frac{-x^2 + 2x + 3}{x^4} = -x^{-2} + 2x^{-3} + 3x^{-4}$ geeft
 $F(x) = x^{-1} - x^{-2} - x^{-3} + c = \frac{1}{x} - \frac{1}{x^2} - \frac{1}{x^3} + c = \frac{x^2 - x - 1}{x^3} + c$
 d $f(x) = \ln(x\sqrt{x}) = \ln(x^{1/2}) = \frac{1}{2}\ln(x)$ geeft $F(x) = \frac{1}{2}(x\ln(x) - x) + c = \frac{1}{2}x\ln(x) - \frac{1}{2}x + c$
 e $f(x) = {}^2\log\left(\frac{1}{x}\right) = {}^2\log(x^{-1}) = -{}^2\log(x)$ geeft
 $F(x) = -\frac{1}{\ln(2)}(x\ln(x) - x) + c = -\frac{x\ln(x) - x}{\ln(2)} + c$
 f $f(x) = 5 \cdot \log(2x) = 5 \cdot \log(2) + 5 \cdot \log(x)$ geeft
 $F(x) = 5 \cdot \log(2) \cdot x + 5 \cdot \frac{1}{\ln(10)} \cdot (x\ln(x) - x) + c = 5x\log(2) + \frac{5x\ln(x) - 5x}{\ln(10)} + c$

- 11** $F_q(x) = (3x + q)x\sqrt{x} = 3x^{3/2} + qx^{1/2}$ geeft $F_q'(x) = \frac{3}{2}x^{1/2} + \frac{1}{2}qx^{-1/2} = \frac{3}{2}x\sqrt{x} + \frac{1}{2}q\sqrt{x}$
 Dit moet gelijk zijn aan $f_p(x) = p\sqrt{x} + 2px\sqrt{x}$, dus $2p = \frac{3}{2}$ en $p = \frac{1}{2}q$
 $p = 3\frac{3}{4}$ en $q = \frac{2}{3}p = \frac{2}{3} \cdot \frac{15}{4} = 2\frac{1}{2}$

Dus voor $p = 3\frac{3}{4}$ en $q = 2\frac{1}{2}$.

- 12** a $f(x) = 2x - 3$ geeft $F(x) = x^2 - 3x + c$
 b $F(1) = 2$ geeft $1 - 3 + c = 2$, dus $c = 4$.
 Dus $F(x) = x^2 - 3x + 4$.
 c De grafiek van $F(x) = x^2 - 3x + c$ raakt de x -as, dus

$$\left. \begin{array}{l} D = 0 \\ D = (-3)^2 - 4 \cdot 1 \cdot c \end{array} \right\} \begin{array}{l} 9 - 4c = 0 \\ -4c = -9 \\ c = 2\frac{1}{4} \end{array}$$

 Dus $F(x) = x^2 - 3x + 2\frac{1}{4}$.

- 13** $f(x) = (x^2 - 1)^2 = x^4 - 2x^2 + 1$ geeft $F(x) = \frac{1}{5}x^5 - \frac{2}{3}x^3 + x + c$
 $F(1) = 7$ geeft $\frac{1}{5} - \frac{2}{3} + 1 + c = 7$, dus $c = 6\frac{7}{15}$.
 Dus $F(x) = \frac{1}{5}x^5 - \frac{2}{3}x^3 + x + 6\frac{7}{15}$.

- 14** $F(x) = (ax^2 + bx + c)e^{2x}$ geeft
 $F'(x) = (2ax + b) \cdot e^{2x} + (ax^2 + bx + c) \cdot e^{2x} \cdot 2 = (2ax^2 + (2a + 2b)x + b + 2c)e^{2x}$
 Dit moet gelijk zijn aan $f(x) = x^2 e^{2x}$, dus $2a = 1 \wedge 2a + 2b = 0 \wedge b + 2c = 0$.
 $2a = 1$ geeft $a = \frac{1}{2}$
 $2a + 2b = 0$ en $a = \frac{1}{2}$ geeft $1 + 2b = 0$, dus $b = -\frac{1}{2}$.
 $b + 2c = 0$ en $b = -\frac{1}{2}$ geeft $-\frac{1}{2} + 2c = 0$, dus $c = \frac{1}{4}$.
 Dus $a = \frac{1}{2}$, $b = -\frac{1}{2}$ en $c = \frac{1}{4}$.

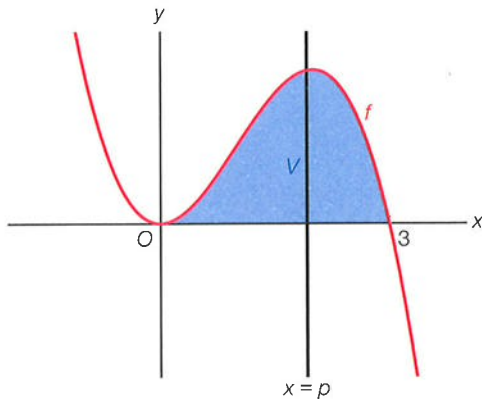
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- 15 a** $A(p) = O(\text{rechthoek}) + O(\text{driehoek}) = p \cdot b + \frac{1}{2}p \cdot (ap + b - b) = bp + \frac{1}{2}p \cdot ap = \frac{1}{2}ap^2 + bp$
b $A'(p) = ap + b$
 Dus $A'(p) = f(p)$.

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- 16 a** $f(x) = 0$ geeft $3x^2 - x^3 = 0$
 $x^2(3 - x) = 0$
 $x = 0 \vee x = 3$

$$O(V) = \int_0^3 (3x^2 - x^3) dx = \left[x^3 - \frac{1}{4}x^4 \right]_0^3 = 3^3 - \frac{1}{4} \cdot 3^4 - 0 = 27 - 20\frac{1}{4} = 6\frac{3}{4}$$

b

$$\int_0^p (3x^2 - x^3) dx = \frac{1}{2} \cdot 6\frac{3}{4}$$

$$\left[x^3 - \frac{1}{4}x^4 \right]_0^p = 3\frac{3}{8}$$

$$p^3 - \frac{1}{4}p^4 - 0 = 3\frac{3}{8}$$

$$p^3 - \frac{1}{4}p^4 = 3\frac{3}{8}$$

Voer in $y_1 = x^3 - \frac{1}{4}x^4$ en $y_2 = 3\frac{3}{8}$.

De optie snijpunt geeft $x \approx 1,84$.

Dus $p \approx 1,84$.

- 17** $O(W) = \frac{1}{2}O(V)$
 $\int_{\frac{1}{3}\pi}^p \sin(x) dx = \frac{1}{2} \cdot \int_0^\pi \sin(x) dx$
 $[-\cos(x)]_{\frac{1}{3}\pi}^p = \frac{1}{2}[-\cos(x)]_0^\pi$
 $-\cos(p) + \cos(\frac{1}{3}\pi) = \frac{1}{2}(-\cos(\pi) + \cos(0))$
 $-\cos(p) + \frac{1}{2} = \frac{1}{2}(1 + 1)$
 $-\cos(p) + \frac{1}{2} = 1$
 $-\cos(p) = \frac{1}{2}$
 $\cos(p) = -\frac{1}{2}$
 $p = \frac{2}{3}\pi$

$$18 \quad a \quad O(V) = \int_0^{12} \sqrt{x} \, dx = \int_0^{12} x^{\frac{1}{2}} \, dx = \left[\frac{2}{3} x^{\frac{3}{2}} \right]_0^{12} = \left[\frac{2}{3} x \sqrt{x} \right]_0^{12} = \frac{2}{3} \cdot 12 \sqrt{12} - 0 = 8\sqrt{12} = 16\sqrt{3}$$

$$b \quad O(V) = 18 \text{ geeft } \int_0^p \sqrt{x} \, dx = 18$$

$$\left[\frac{2}{3} x \sqrt{x} \right]_0^p = 18$$

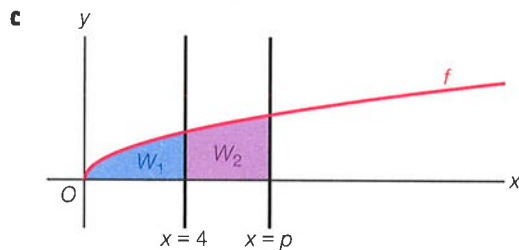
$$\frac{2}{3} p \sqrt{p} = 18$$

$$p \sqrt{p} = 27$$

$$p^{1\frac{1}{2}} = 3^3$$

$$p = 3^2$$

$$p = 9$$



$$O(W_1) = O(W_2)$$

$$\int_0^4 \sqrt{x} \, dx = \int_4^p \sqrt{x} \, dx$$

$$\left[\frac{2}{3} x \sqrt{x} \right]_0^4 = \left[\frac{2}{3} x \sqrt{x} \right]_4^p$$

$$\frac{2}{3} \cdot 4 \sqrt{4} - 0 = \frac{2}{3} p \sqrt{p} - \frac{2}{3} \cdot 4 \sqrt{4}$$

$$\frac{2}{3} p \sqrt{p} = 2 \cdot \frac{2}{3} \cdot 4 \sqrt{4}$$

$$p^{1\frac{1}{2}} = 16$$

$$p = 16^{\frac{2}{3}}$$

$$p = (2^4)^{\frac{2}{3}}$$

$$p = 2^{\frac{8}{3}}$$

$$p = 4 \cdot \sqrt[3]{4}$$

Bladzijde 106

$$19 \quad a \quad O(V) = 5$$

$$\int_0^p e^x \, dx = 5$$

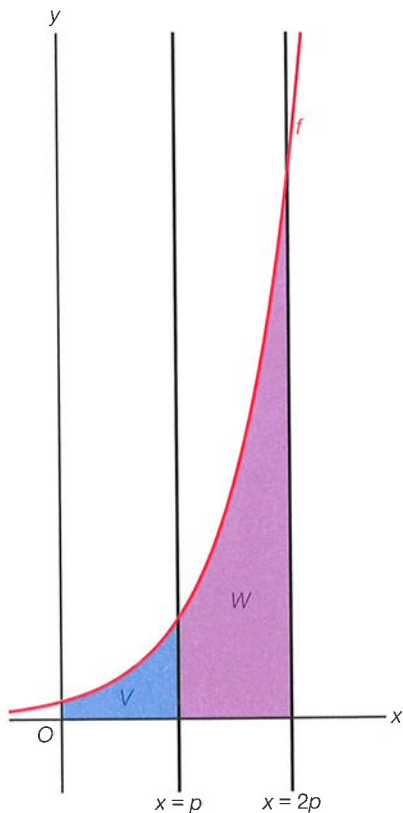
$$\left[e^x \right]_0^p = 5$$

$$e^p - e^0 = 5$$

$$e^p - 1 = 5$$

$$e^p = 6$$

$$p = \ln(6)$$

b

$$O(W) = 2 \cdot O(V)$$

$$\int_p^{2p} e^x dx = 2 \int_0^p e^x dx$$

$$[e^x]_p^{2p} = 2[e^x]_0^p$$

$$e^{2p} - e^p = 2(e^p - e^0)$$

$$e^{2p} - e^p = 2e^p - 2$$

$$(e^p)^2 - 3e^p + 2 = 0$$

$$\text{Stel } e^p = u.$$

$$u^2 - 3u + 2 = 0$$

$$(u-1)(u-2) = 0$$

$$u = 1 \vee u = 2$$

$$e^p = 1 \vee e^p = 2$$

$$p = 0 \vee p = \ln(2)$$

vold. niet

Dus $p = \ln(2)$.

20 a $O(V) = \int_1^{e^2} \ln(x) dx = [x \ln(x) - x]_1^{e^2} = e^2 \ln(e^2) - e^2 - (\ln(1) - 1) = 2e^2 - e^2 + 1 = e^2 + 1$

b $O(V) = 10$

$$\int_1^p \ln(x) dx = 10$$

$$[x \ln(x) - x]_1^p = 10$$

$$p \ln(p) - p - (\ln(1) - 1) = 10$$

$$p \ln(p) - p + 1 = 10$$

$$p \ln(p) - p = 9$$

Voer in $y_1 = x \ln(x) - x$ en $y_2 = 9$.

De optie snijpunt geeft $x \approx 8,174$.

Dus $p \approx 8,174$.

21 a $F_p(x) = \frac{1}{3}p \cos^3(x) - p \cos(x)$ geeft

$$F_p'(x) = p \cos^2(x) \cdot -\sin(x) - p \cdot -\sin(x) = -p \cos^2(x) \sin(x) + p \sin(x) \\ = p \sin(x)(-\cos^2(x) + 1) = p \sin(x) \cdot \sin^2(x) = p \sin^3(x)$$

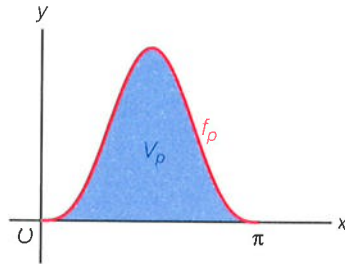
Dus $F_p(x) = \frac{1}{3}p \cos^3(x) - p \cos(x)$ is een primitieve van $f_p(x) = p \sin^3(x)$.

b $f_p(x) = 0$ geeft $p \sin^3(x) = 0$

$$\sin(x) = 0$$

$$x = 0 + k \cdot \pi$$

x in $[0, \pi]$ geeft $x = 0 \vee x = \pi$



$$O(V_p) = \int_0^\pi f_p(x) dx = \left[\frac{1}{3}p \cos^3(x) - p \cos(x) \right]_0^\pi \\ = \frac{1}{3}p \cdot (-1)^3 - p \cdot -1 - \left(\frac{1}{3}p \cdot 1^3 - p \cdot 1 \right) = -\frac{1}{3}p + p - \frac{1}{3}p + p = 1\frac{1}{3}p$$

$$O(V_p) = 4 \text{ geeft } 1\frac{1}{3}p = 4 \\ p = 3$$

11.2 Oppervlakten

Bladzijde 108

22 a $F(x) = a(3x+1)^6 + c$ geeft $F'(x) = a \cdot 6(3x+1)^5 \cdot 3 = 18a(3x+1)^5$

b $F'(x) = 18a(3x+1)^5$ moet gelijk zijn aan $f(x) = (3x+1)^5$, dus $18a = 1$ en dit geeft $a = \frac{1}{18}$.

Bladzijde 109

23 $\left[\frac{1}{a} F(ax+b) + c \right]' = \frac{1}{a} \cdot f(ax+b) \cdot a = f(ax+b)$

Dus $\frac{1}{a} F(ax+b) + c$ zijn de primitieven van $f(ax+b)$.

24 a $f(x) = (ax+b)^n$ geeft $F(x) = \frac{1}{a} \cdot \frac{1}{n+1} (ax+b)^{n+1} + c$

b $f(x) = g^{ax+b}$ geeft $F(x) = \frac{1}{a} \cdot \frac{g^{ax+b}}{\ln(g)} + c$

c $f(x) = \frac{1}{ax+b}$ geeft $F(x) = \frac{1}{a} \ln|ax+b| + c$

d $f(x) = \ln(ax+b)$ geeft $F(x) = \frac{1}{a} ((ax+b) \ln(ax+b) - (ax+b)) + c$

e $f(x) = {}^g\log(ax+b)$ geeft $F(x) = \frac{1}{a} \cdot \frac{1}{\ln(g)} ((ax+b) \ln(ax+b) - (ax+b)) + c$

f $f(x) = \cos(ax+b)$ geeft $F(x) = \frac{1}{a} \sin(ax+b) + c$

25 a $f(x) = (2x-1)^6$ geeft $F(x) = \frac{1}{2} \cdot \frac{1}{7} (2x-1)^7 + c = \frac{1}{14} (2x-1)^7 + c$

b $f(x) = \frac{1}{(3x+4)^3} = (3x+4)^{-3}$ geeft $F(x) = \frac{1}{3} \cdot -\frac{1}{2} (3x+4)^{-2} + c = -\frac{1}{6(3x+4)^2} + c$

c $f(x) = 4\sqrt{3-2x} = 4(3-2x)^{\frac{1}{2}}$ geeft $F(x) = 4 \cdot -\frac{1}{2} \cdot \frac{2}{3} (3-2x)^{\frac{3}{2}} + c = -\frac{4}{3} (3-2x)\sqrt{3-2x} + c$

d $f(x) = \frac{2}{\sqrt{1-x}} = 2(1-x)^{-\frac{1}{2}}$ geeft $F(x) = 2 \cdot -2(1-x)^{\frac{1}{2}} + c = -4\sqrt{1-x} + c$

e $f(x) = \sin(2x + \frac{1}{3}\pi)$ geeft $F(x) = -\frac{1}{2} \cos(2x + \frac{1}{3}\pi) + c$

f $f(x) = 3 \cos(\frac{1}{2}x - \frac{1}{6}\pi)$ geeft $F(x) = 3 \cdot 2 \sin(\frac{1}{2}x - \frac{1}{6}\pi) + c = 6 \sin(\frac{1}{2}x - \frac{1}{6}\pi) + c$

- 26 a** $f(x) = \frac{1}{x-1}$ geeft $F(x) = \ln|x-1| + c$
- b** $f(x) = (2x+1)\sqrt{2x+1} = (2x+1)^{1\frac{1}{2}}$ geeft
 $F(x) = \frac{1}{2} \cdot \frac{2}{5} (2x+1)^{2\frac{1}{2}} + c = \frac{1}{5} (2x+1)^2 \cdot \sqrt{2x+1} + c$
- c** $f(x) = e^{4x-1}$ geeft $F(x) = \frac{1}{4} e^{4x-1} + c$
- d** $f(x) = \ln(4x-1)$ geeft
 $F(x) = \frac{1}{4} ((4x-1)\ln(4x-1) - (4x-1)) + c = (x - \frac{1}{4})\ln(4x-1) - (x - \frac{1}{4}) + c$
- e** $f(x) = \frac{3}{2x-5}$ geeft $F(x) = 3 \cdot \frac{1}{2} \ln|2x-5| + c = \frac{3}{2} \ln|2x-5| + c$
- f** $f(x) = 2^{3x}$ geeft $F(x) = \frac{1}{3} \cdot \frac{2^{3x}}{\ln(2)} + c = \frac{2^{3x}}{3 \ln(2)} + c$
- g** $f(x) = 3^{2-5x}$ geeft $F(x) = -\frac{1}{5} \cdot \frac{3^{2-5x}}{\ln(3)} + c = -\frac{3^{2-5x}}{5 \ln(3)} + c$
- h** $f(x) = {}^2\log(5x+3)$ geeft
 $F(x) = \frac{1}{5} \cdot \frac{1}{\ln(2)} \cdot ((5x+3)\ln(5x+3) - (5x+3)) + c = \frac{(5x+3)\ln(5x+3) - (5x+3)}{5 \ln(2)} + c$

- 27 a** $f(x) = \frac{3}{(2x-1)^4} = 3(2x-1)^{-4}$ geeft $F(x) = 3 \cdot \frac{1}{2} \cdot -\frac{1}{3} (2x-1)^{-3} + c = -\frac{1}{2(2x-1)^3} + c$
- b** $f(x) = (4x+3)\sqrt{4x+3} = (4x+3)^{1\frac{1}{2}}$ geeft $F(x) = \frac{1}{4} \cdot \frac{2}{5} (4x+3)^{2\frac{1}{2}} + c = \frac{1}{10} (4x+3)^2 \cdot \sqrt{4x+3} + c$
- c** $f(x) = 2 \sin(3x + \frac{1}{2}\pi)$ geeft $F(x) = 2 \cdot -\frac{1}{3} \cos(3x + \frac{1}{2}\pi) + c = -\frac{2}{3} \cos(3x + \frac{1}{2}\pi) + c$
- d** $f(x) = x^2 - \ln(\frac{1}{2}x + 1)$ geeft
 $F(x) = \frac{1}{3}x^3 - 2((\frac{1}{2}x+1)\ln(\frac{1}{2}x+1) - (\frac{1}{2}x+1)) + c = \frac{1}{3}x^3 - (x+2)\ln(\frac{1}{2}x+1) + x+2 + c$
- e** $f(x) = \frac{6}{3x+1}$ geeft $F(x) = 6 \cdot \frac{1}{3} \cdot \ln|3x+1| + c = 2 \ln|3x+1| + c$
- f** $f(x) = e^{4x+1} - 4^{ex}$ geeft $F(x) = \frac{1}{4} e^{4x+1} - \frac{1}{e} \cdot \frac{4^{ex}}{\ln(4)} + c = \frac{1}{4} e^{4x+1} - \frac{4^{ex}}{e \ln(4)} + c$
- g** $f(x) = 5 \cdot \log(3x+2)$ geeft
 $F(x) = 5 \cdot \frac{1}{3} \cdot \frac{1}{\ln(10)} \cdot ((3x+2)\ln(3x+2) - (3x+2)) + c = \frac{5((3x+2)\ln(3x+2) - (3x+2))}{3 \ln(10)} + c$
- h** $f(x) = 3x - \cos(3x+1)$ geeft $F(x) = \frac{1}{2}x^2 - \frac{1}{3} \sin(3x+1) + c$

- 28 a** $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} (2 \sin(x - \frac{1}{6}\pi)) dx = [-2 \cos(x - \frac{1}{6}\pi)]_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} = -2 \cos(\frac{1}{6}\pi) + 2 \cos(0) = -\sqrt{3} + 2$
- b** $\int_0^{\frac{1}{3}\pi} (2x + \cos(\frac{1}{2}x)) dx = [x^2 + 2 \sin(\frac{1}{2}x)]_0^{\frac{1}{3}\pi} = (\frac{1}{3}\pi)^2 + 2 \sin(\frac{1}{6}\pi) - (0 + 0) = \frac{1}{9}\pi^2 + 2 \cdot \frac{1}{2} = \frac{1}{9}\pi^2 + 1$

29 $f(x) = 0$ geeft $1 + 2 \cos(\frac{1}{2}x - \frac{5}{6}\pi) = 0$

$$2 \cos(\frac{1}{2}x - \frac{5}{6}\pi) = -1$$

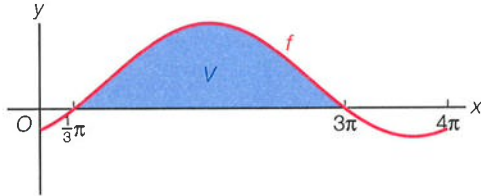
$$\cos(\frac{1}{2}x - \frac{5}{6}\pi) = -\frac{1}{2}$$

$$\frac{1}{2}x - \frac{5}{6}\pi = \frac{2}{3}\pi + k \cdot 2\pi \vee \frac{1}{2}x - \frac{5}{6}\pi = -\frac{2}{3}\pi + k \cdot 2\pi$$

$$\frac{1}{2}x = 1\frac{1}{2}\pi + k \cdot 2\pi \vee \frac{1}{2}x = \frac{1}{6}\pi + k \cdot 2\pi$$

$$x = 3\pi + k \cdot 4\pi \vee x = \frac{1}{3}\pi + k \cdot 4\pi$$

x in $[0, 4\pi]$ geeft $x = \frac{1}{3}\pi \vee x = 3\pi$



$$\begin{aligned} O(V) &= \int_{\frac{1}{3}\pi}^{3\pi} (1 + 2 \cos(\frac{1}{2}x - \frac{5}{6}\pi)) dx = \left[x + 4 \sin(\frac{1}{2}x - \frac{5}{6}\pi) \right]_{\frac{1}{3}\pi}^{3\pi} = 3\pi + 4 \sin(\frac{2}{3}\pi) - (\frac{1}{3}\pi + 4 \sin(-\frac{2}{3}\pi)) \\ &= 3\pi + 4 \cdot \frac{1}{2}\sqrt{3} - (\frac{1}{3}\pi + 4 \cdot -\frac{1}{2}\sqrt{3}) = 3\pi + 2\sqrt{3} - \frac{1}{3}\pi + 2\sqrt{3} = 2\frac{2}{3}\pi + 4\sqrt{3} \end{aligned}$$

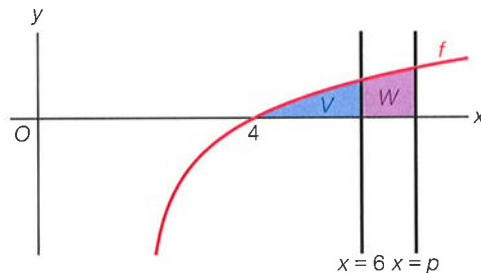
Bladzijde 110

30 $f(x) = 0$ geeft $\ln(\frac{1}{2}x - 1) = 0$

$$\frac{1}{2}x - 1 = 1$$

$$\frac{1}{2}x = 2$$

$$x = 4$$



$$f(x) = \ln(\frac{1}{2}x - 1) \text{ geeft } F(x) = 2((\frac{1}{2}x - 1) \ln(\frac{1}{2}x - 1) - (\frac{1}{2}x - 1)) = (x - 2) \ln(\frac{1}{2}x - 1) - x + 2$$

$$O(V) = \int_4^6 f(x) dx = \left[(x - 2) \ln(\frac{1}{2}x - 1) - x + 2 \right]_4^6 = 4 \ln(2) - 6 + 2 - (2 \ln(1) - 4 + 2) = 4 \ln(2) - 2$$

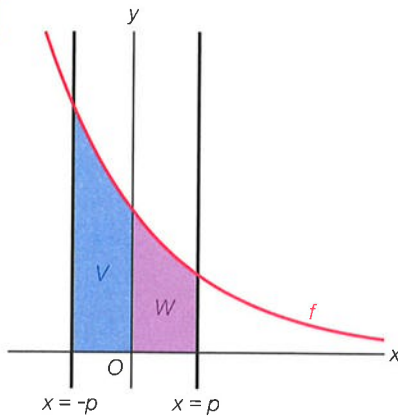
$$\begin{aligned} O(W) &= \int_6^p f(x) dx = \left[(x - 2) \ln(\frac{1}{2}x - 1) - x + 2 \right]_6^p = (p - 2) \ln(\frac{1}{2}p - 1) - p + 2 - (4 \ln(2) - 4) \\ &= (p - 2) \ln(\frac{1}{2}p - 1) - p + 6 - 4 \ln(2) \end{aligned}$$

Voer in $y_1 = 4 \ln(2) - 2$ en $y_2 = (x - 2) \ln(\frac{1}{2}x - 1) - x + 6 - 4 \ln(2)$.

De optie snijpunt geeft $x = 2,119...$ en $x = 6,960...$

Dus $p \approx 6,96$.

31



$$O(V) = 2O(W)$$

$$\int_{-p}^0 2e^{1-\frac{1}{3}x} dx = 2 \int_0^p 2e^{1-\frac{1}{3}x} dx$$

$$\left[2 \cdot -3e^{1-\frac{1}{3}x} \right]_{-p}^0 = 2 \left[2 \cdot -3e^{1-\frac{1}{3}x} \right]_0^p$$

$$\left[-6e^{1-\frac{1}{3}x} \right]_{-p}^0 = 2 \left[-6e^{1-\frac{1}{3}x} \right]_0^p$$

$$-6e^1 + 6e^{1+\frac{1}{3}p} = 2(-6e^{1-\frac{1}{3}p} + 6e^1)$$

$$-6e + 6e \cdot e^{\frac{1}{3}p} = -12e \cdot e^{-\frac{1}{3}p} + 12e$$

$$-1 + e^{\frac{1}{3}p} = -2e^{-\frac{1}{3}p} + 2$$

$$\text{Stel } e^{\frac{1}{3}p} = u.$$

$$-1 + u = -\frac{2}{u} + 2$$

$$-u + u^2 = -2 + 2u$$

$$u^2 - 3u + 2 = 0$$

$$(u-1)(u-2) = 0$$

$$u = 1 \vee u = 2$$

$$e^{\frac{1}{3}p} = 1 \vee e^{\frac{1}{3}p} = 2$$

$$\frac{1}{3}p = 0 \vee \frac{1}{3}p = \ln(2)$$

$$p = 0 \vee p = 3 \ln(2)$$

vold. niet

$$\text{Dus } p = 3 \ln(2).$$

32 a De regel gaat over functies van de vorm $f(ax+b)$ en g is van de vorm $f(ax^2+b)$.

b $G(x) = a(4x^2 - 1)^{\frac{1}{2}}$ geeft $G'(x) = \frac{1}{2}a(4x^2 - 1)^{-\frac{1}{2}} \cdot 8x = 4ax\sqrt{4x^2 - 1}$

Er bestaat geen waarde van a waarvoor $G'(x) = \sqrt{4x^2 - 1}$.

33 a Stel A is het gebied begrensd door de x -as, de lijnen $x = a$ en $x = b$ en de grafiek van f .

Dit geeft $O(A) = \int_a^b f(x) dx$.

Stel B is het gebied begrensd door de x -as, de lijnen $x = a$ en $x = b$ en de grafiek van g .

Dit geeft $O(B) = \int_a^b g(x) dx$.

Dus $O(V) = O(A) - O(B) = \int_a^b f(x) dx - \int_a^b g(x) dx$.

b $O(V) = \int_a^b f(x) dx - \int_a^b g(x) dx = [F(x)]_a^b - [G(x)]_a^b = F(b) - F(a) - (G(b) - G(a))$
 $= F(b) - F(a) - G(b) + G(a) = F(b) - G(b) - F(a) + G(a)$
 $= F(b) - G(b) - (F(a) - G(a))$

c $O(V) = F(b) - G(b) - (F(a) - G(a)) = [F(x) - G(x)]_a^b = \int_a^b (f(x) - g(x)) dx$

Bladzijde 112

- 34** Verschuif de grafieken van f en g c omhoog waarbij c voldoende groot is om V geheel boven de x -as te krijgen. De beeldgrafieken zijn dan $y = f(x) + c$ en $y = g(x) + c$.

$$\text{Dus } O(V) = \int_a^b (f(x) + c - (g(x) + c)) dx = \int_a^b (f(x) + c - g(x) - c) dx = \int_a^b (f(x) - g(x)) dx.$$

35 a $O(V) = \int_0^{\pi} \sin(x) dx = [-\cos(x)]_0^{\pi} = -\cos(\pi) + \cos(0) = 1 + 1 = 2$

$$O(W) = -\int_{\pi}^{2\pi} \sin(x) dx = -[-\cos(x)]_{\pi}^{2\pi} = [\cos(x)]_{\pi}^{2\pi} = \cos(2\pi) - \cos(\pi) = 1 - (-1) = 2$$

b $\int_0^{2\pi} \sin(x) dx = [-\cos(x)]_0^{2\pi} = -\cos(2\pi) + \cos(0) = -1 + 1 = 0$

$$\int_0^{2\pi} \sin(x) dx = \int_0^{\pi} \sin(x) dx + \int_{\pi}^{2\pi} \sin(x) dx = O(V) + -O(W) = 2 - 2 = 0$$

Bladzijde 113

- 36** $f(x) = g(x)$ geeft $\sin(x) = \cos(2x)$

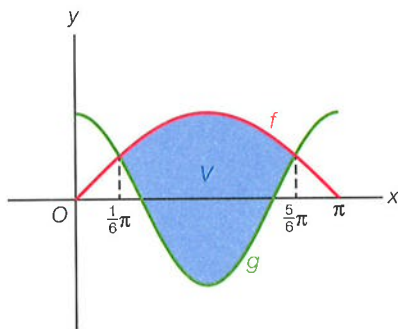
$$\cos(x - \frac{1}{2}\pi) = \cos(2x)$$

$$x - \frac{1}{2}\pi = 2x + k \cdot 2\pi \vee x - \frac{1}{2}\pi = -2x + k \cdot 2\pi$$

$$-x = \frac{1}{2}\pi + k \cdot 2\pi \vee 3x = \frac{1}{2}\pi + k \cdot 2\pi$$

$$x = -\frac{1}{2}\pi + k \cdot 2\pi \vee x = \frac{1}{6}\pi + k \cdot \frac{2}{3}\pi$$

$$x \text{ in } [0, \pi] \text{ geeft } x = \frac{1}{6}\pi \vee x = \frac{5}{6}\pi$$



$$\begin{aligned} O(V) &= \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (f(x) - g(x)) dx = \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (\sin(x) - \cos(2x)) dx = [-\cos(x) - \frac{1}{2}\sin(2x)]_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} \\ &= -\cos(\frac{5}{6}\pi) - \frac{1}{2}\sin(\frac{5}{3}\pi) - (-\cos(\frac{1}{6}\pi) - \frac{1}{2}\sin(\frac{1}{3}\pi)) = \frac{1}{2}\sqrt{3} - \frac{1}{2} \cdot -\frac{1}{2}\sqrt{3} - (-\frac{1}{2}\sqrt{3} - \frac{1}{2} \cdot \frac{1}{2}\sqrt{3}) \\ &= \frac{1}{2}\sqrt{3} + \frac{1}{4}\sqrt{3} + \frac{1}{2}\sqrt{3} + \frac{1}{4}\sqrt{3} = 1\frac{1}{2}\sqrt{3} \end{aligned}$$

- 37 a** $x^3 = 2x$

$$x = 0 \vee x^2 = 2$$

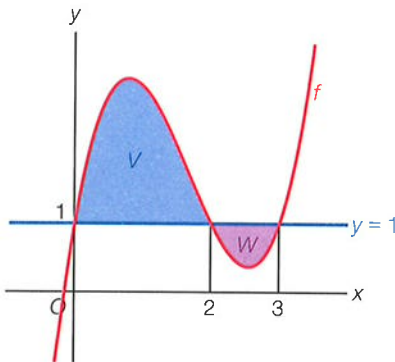
$$x = 0 \vee x = \sqrt{2} \vee x = -\sqrt{2}$$

$$O(V) = \int_0^{\sqrt{2}} (2x - x^3) dx = [x^2 - \frac{1}{4}x^4]_0^{\sqrt{2}} = 2 - \frac{1}{4} \cdot 4 - 0 = 1$$

$$\begin{aligned}
 \text{b } \int_0^p (2x - x^3) dx &= \frac{1}{2} \text{ met } 0 < p < \sqrt{2} \\
 \left[x^2 - \frac{1}{4}x^4 \right]_0^p &= \frac{1}{2} \\
 p^2 - \frac{1}{4}p^4 - 0 &= \frac{1}{2} \\
 4p^2 - p^4 &= 2 \\
 p^4 - 4p^2 + 2 &= 0 \\
 \text{Stel } p^2 &= u. \\
 u^2 - 4u + 2 &= 0 \\
 (u - 2)^2 - 4 + 2 &= 0 \\
 (u - 2)^2 &= 2 \\
 u - 2 &= \sqrt{2} \vee u - 2 = -\sqrt{2} \\
 u &= 2 + \sqrt{2} \vee u = 2 - \sqrt{2} \\
 p^2 &= 2 + \sqrt{2} \vee p^2 = 2 - \sqrt{2} \\
 \text{vold. niet} \quad p &= \sqrt{2 + \sqrt{2}} \vee p = -\sqrt{2 + \sqrt{2}} \\
 &\quad \text{vold. niet} \\
 \text{Dus } p &= \sqrt{2 - \sqrt{2}}.
 \end{aligned}$$

38 $f(x) = 1$ geeft $x^3 - 5x^2 + 6x + 1 = 1$

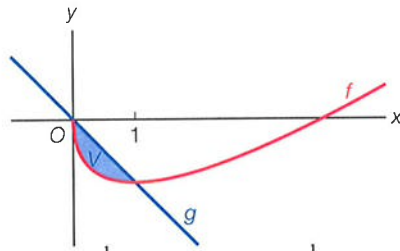
$$\begin{aligned}
 x^3 - 5x^2 + 6x &= 0 \\
 x(x^2 - 5x + 6) &= 0 \\
 x(x - 2)(x - 3) &= 0 \\
 x &= 0 \vee x = 2 \vee x = 3
 \end{aligned}$$



$$\begin{aligned}
 O(V) &= \int_0^2 (f(x) - 1) dx = \int_0^2 (x^3 - 5x^2 + 6x) dx = \left[\frac{1}{4}x^4 - \frac{5}{3}x^3 + 3x^2 \right]_0^2 \\
 &= \frac{1}{4} \cdot 2^4 - \frac{5}{3} \cdot 2^3 + 3 \cdot 2^2 - 0 = 2\frac{2}{3} \\
 O(W) &= \int_2^3 (1 - f(x)) dx = \int_2^3 (1 - (x^3 - 5x^2 + 6x + 1)) dx = \int_2^3 (1 - x^3 + 5x^2 - 6x - 1) dx \\
 &= \int_2^3 (-x^3 + 5x^2 - 6x) dx = \left[-\frac{1}{4}x^4 + \frac{5}{3}x^3 - 3x^2 \right]_2^3 \\
 &= -\frac{1}{4} \cdot 3^4 + \frac{5}{3} \cdot 3^3 - 3 \cdot 3^2 - \left(-\frac{1}{4} \cdot 2^4 + \frac{5}{3} \cdot 2^3 - 3 \cdot 2^2 \right) = \frac{5}{12} \\
 \frac{O(V)}{O(W)} &= \frac{2\frac{2}{3}}{\frac{5}{12}} = \frac{8}{3} \cdot \frac{12}{5} = \frac{32}{5}
 \end{aligned}$$

Dus $O(V)$ is $6\frac{2}{5}$ keer zo groot als $O(W)$.

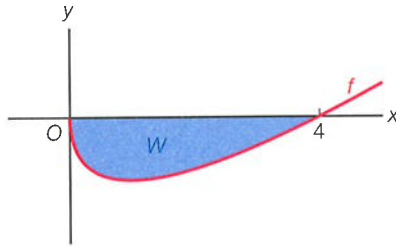
39 $f(x) = g(x)$ geeft $x - 2\sqrt{x} = -x$
 $2x = 2\sqrt{x}$
 $x = \sqrt{x}$
kwadrateren geeft
 $x^2 = x$
 $x = 0 \vee x = 1$



$$O(V) = \int_0^1 (g(x) - f(x)) dx = \int_0^1 (-x - (x - 2\sqrt{x})) dx = \int_0^1 (-2x + 2\sqrt{x}) dx = \int_0^1 (-2x + 2x^{\frac{1}{2}}) dx$$

$$= \left[-x^2 + 2 \cdot \frac{2}{3} x^{\frac{3}{2}} \right]_0^1 = \left[-x^2 + \frac{4}{3} x\sqrt{x} \right]_0^1 = -1^2 + \frac{4}{3} \cdot 1 \cdot \sqrt{1} - 0 = -1 + \frac{4}{3} = \frac{1}{3}$$

$f(x) = 0$ geeft $x - 2\sqrt{x} = 0$
 $x = 2\sqrt{x}$
kwadrateren geeft
 $x^2 = 4x$
 $x = 0 \vee x = 4$



$$O(W) = - \int_0^4 f(x) dx = - \int_0^4 (x - 2\sqrt{x}) dx = \int_0^4 (-x + 2x^{\frac{1}{2}}) dx = \left[-\frac{1}{2}x^2 + 2 \cdot \frac{2}{3} x^{\frac{3}{2}} \right]_0^4 = \left[-\frac{1}{2}x^2 + \frac{4}{3} x\sqrt{x} \right]_0^4$$

$$= -\frac{1}{2} \cdot 4^2 + \frac{4}{3} \cdot 4 \cdot \sqrt{4} - 0 = -8 + \frac{16}{3} \cdot 2 = -8 + \frac{32}{3} = \frac{8}{3}$$

$$8 \cdot O(V) = 8 \cdot \frac{1}{3} = \frac{8}{3} = O(W)$$

40 a $f(x) = -1\frac{1}{2}$ geeft $\frac{x^2 + x + 1}{x} = -1\frac{1}{2}$
 $x^2 + x + 1 = -1\frac{1}{2}x$
 $x^2 + 2\frac{1}{2}x + 1 = 0$
 $(x + \frac{1}{2})(x + 2) = 0$
 $x = -\frac{1}{2} \vee x = -2$

$$O(V) = \int_{-2}^{-\frac{1}{2}} \left(\frac{x^2 + x + 1}{x} - (-1\frac{1}{2}) \right) dx = \int_{-2}^{-\frac{1}{2}} \left(x + 1 + \frac{1}{x} + 1\frac{1}{2} \right) dx = \int_{-2}^{-\frac{1}{2}} \left(x + \frac{1}{x} + 2\frac{1}{2} \right) dx$$

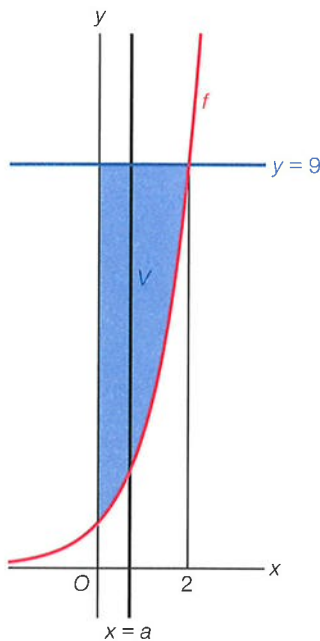
$$= \left[\frac{1}{2}x^2 + \ln|x| + 2\frac{1}{2}x \right]_{-2}^{-\frac{1}{2}} = \frac{1}{8} + \ln(\frac{1}{2}) - 1\frac{1}{4} - (2 + \ln(2) - 5) = \frac{1}{8} + \ln(\frac{1}{2}) - 1\frac{1}{4} - 2 - \ln(2) + 5$$

$$= 1\frac{7}{8} + \ln(\frac{1}{2}) - \ln(2) = 1\frac{7}{8} + \ln(\frac{1}{4})$$

$$\begin{aligned}
 \text{b } O(W) = 2 \text{ geeft } \int_1^p \left(\frac{x^2 + x + 1}{x} - (x + 1) \right) dx &= 2 \\
 \int_1^p \left(x + 1 + \frac{1}{x} - (x + 1) \right) dx &= 2 \\
 \int_1^p \frac{1}{x} dx &= 2 \\
 [\ln|x|]_1^p &= 2 \\
 \ln|p| - \ln(1) &= 2 \\
 \ln(p) &= 2 \\
 p &= e^2
 \end{aligned}$$

Bladzijde 114

41 $f(x) = 9$ geeft $3^x = 9$
 $x = 2$



$$O(V) = \int_0^2 (9 - 3^x) dx = \left[9x - \frac{3^x}{\ln(3)} \right]_0^2 = 18 - \frac{9}{\ln(3)} - \left(0 - \frac{1}{\ln(3)} \right) = 18 - \frac{8}{\ln(3)}$$

$$O(W) = \int_0^a (9 - 3^x) dx = \left[9x - \frac{3^x}{\ln(3)} \right]_0^a = 9a - \frac{3^a}{\ln(3)} - \left(0 - \frac{1}{\ln(3)} \right) = 9a + \frac{1 - 3^a}{\ln(3)}$$

Los op $9a + \frac{1 - 3^a}{\ln(3)} = 9 - \frac{4}{\ln(3)}$.

Voer in $y_1 = 9x + \frac{1 - 3^x}{\ln(3)}$ en $y_2 = 9 - \frac{4}{\ln(3)}$.

De optie snijpunt geeft $x = 0,716\dots$ en $x = 2,875\dots$

Dus $a \approx 0,72$.

42 $f(x) = 8$ geeft $\frac{8}{x^2} = 8$
 $x^2 = 1$
 $x = 1 \vee x = -1$
 vold. niet

$$O(V) = 1 \cdot 8 + \int_1^8 \frac{8}{x^2} dx = 8 + \int_1^8 8x^{-2} dx = 8 + [-8x^{-1}]_1^8 = 8 + \left[-\frac{8}{x}\right]_1^8 = 8 - \frac{8}{8} + \frac{8}{1} = 15$$

$$O(V_1) = \frac{2}{3} O(V) \text{ geeft } 8 + \int_1^a \frac{8}{x^2} dx = \frac{2}{3} \cdot 15$$

$$8 + \int_1^a 8x^{-2} dx = 10$$

$$\int_1^a 8x^{-2} dx = 2$$

$$\left[-\frac{8}{x}\right]_1^a = 2$$

$$-\frac{8}{a} + \frac{8}{1} = 2$$

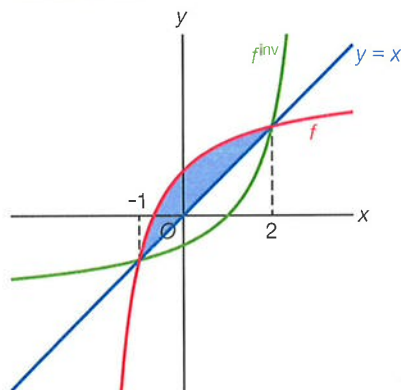
$$-\frac{8}{a} = -6$$

$$-6a = -8$$

$$a = 1\frac{1}{3}$$

Bladzijde 115

43



De snijpunten van de grafiek f en de grafiek van f^{inv} liggen op de lijn $y = x$.

$$f(x) = x \text{ geeft } 3 - \frac{4}{x+2} = x$$

$$3(x+2) - 4 = x(x+2)$$

$$3x + 6 - 4 = x^2 + 2x$$

$$x^2 - x - 2 = 0$$

$$(x+1)(x-2) = 0$$

$$x = -1 \vee x = 2$$

$$O(V) = 2 \cdot \int_{-1}^2 (f(x) - x) dx = 2 \cdot \int_{-1}^2 \left(3 - \frac{4}{x+2} - x\right) dx = 2 \cdot \left[3x - 4 \ln|x+2| - \frac{1}{2}x^2\right]_{-1}^2$$

$$= 2(6 - 4 \ln(4) - 2 - 2(-3 - 4 \ln(1) - \frac{1}{2})) = 8 - 8 \ln(4) + 7 = 15 - 8 \ln(4)$$

44 $P(p, 2e^{0,2p})$
 $O_{\text{rechthoek}} = p \cdot 2e^{0,2p}$
 $O(V) = O(W)$
 $O(W) = \frac{1}{2}O_{\text{rechthoek}}$

$$\int_0^p (2e^{0,2x}) dx = p \cdot e^{0,2p}$$

$$[10e^{0,2x}]_0^p = p \cdot e^{0,2p}$$

$$10e^{0,2p} - 10 = p \cdot e^{0,2p}$$

Voer in $y_1 = 10e^{0,2x} - 10$ en $y_2 = x \cdot e^{0,2x}$.

De optie snijpunt geeft $x = 7,968...$

Dus $p \approx 7,97$.

45 a $F_p(x) = -2p\sqrt{1-x}$ geeft $F_p'(x) = -2p \cdot \frac{1}{2\sqrt{1-x}} \cdot -1 = \frac{p}{\sqrt{1-x}}$

Dus $F_p(x) = -2p\sqrt{1-x}$ is een primitieve van $f_p(x) = \frac{p}{\sqrt{1-x}}$.

b $O(V_q) = \int_q^0 (f_6(x) - f_2(x)) dx = \int_q^0 \left(\frac{6}{\sqrt{1-x}} - \frac{2}{\sqrt{1-x}} \right) dx = \int_q^0 \left(\frac{4}{\sqrt{1-x}} \right) dx$
 $= [-8\sqrt{1-x}]_q^0 = -8\sqrt{1} + 8\sqrt{1-q} = -8 + 8\sqrt{1-q}$

$$O(V_q) = 12 \text{ geeft } -8 + 8\sqrt{1-q} = 12$$

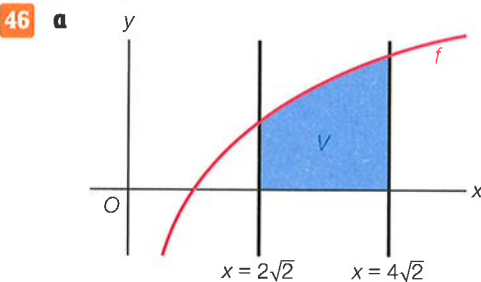
$$8\sqrt{1-q} = 20$$

$$\sqrt{1-q} = 2\frac{1}{2}$$

$$1-q = 6\frac{1}{4}$$

$$q = -5\frac{1}{4}$$

Bladzijde 116



$$O(V) = \int_{2\sqrt{2}}^{4\sqrt{2}} \ln(x) dx = [x \ln(x) - x]_{2\sqrt{2}}^{4\sqrt{2}} = 4\sqrt{2} \cdot \ln(4\sqrt{2}) - 4\sqrt{2} - (2\sqrt{2} \cdot \ln(2\sqrt{2}) - 2\sqrt{2})$$

$$= 4\sqrt{2} \cdot 2\frac{1}{2} \ln(2) - 4\sqrt{2} - 2\sqrt{2} \cdot 1\frac{1}{2} \ln(2) + 2\sqrt{2} = 10\sqrt{2} \cdot \ln(2) - 2\sqrt{2} - 3\sqrt{2} \cdot \ln(2)$$

$$= 7\sqrt{2} \cdot \ln(2) - 2\sqrt{2} = \sqrt{2} \cdot (7 \ln(2) - 2)$$

b $L = \int_{2\sqrt{2}}^{4\sqrt{5}} \sqrt{1 + (f'(x))^2} dx = \int_{2\sqrt{2}}^{4\sqrt{5}} \sqrt{1 + \frac{1}{x^2}} dx = \int_{2\sqrt{2}}^{4\sqrt{5}} \sqrt{\frac{x^2 + 1}{x^2}} dx = \int_{2\sqrt{2}}^{4\sqrt{5}} \frac{\sqrt{x^2 + 1}}{x} dx$

$$= \left[\sqrt{x^2 + 1} - \ln\left(\frac{1 + \sqrt{x^2 + 1}}{x}\right) \right]_{2\sqrt{2}}^{4\sqrt{5}} = \sqrt{80 + 1} - \ln\left(\frac{1 + \sqrt{80 + 1}}{4\sqrt{5}}\right) - \left(\sqrt{8 + 1} - \ln\left(\frac{1 + \sqrt{8 + 1}}{2\sqrt{2}}\right) \right)$$

$$= 9 - \ln\left(\frac{10}{4\sqrt{5}}\right) - \left(3 - \ln\left(\frac{4}{2\sqrt{2}}\right) \right) = 6 - \ln\left(\frac{1}{2}\sqrt{5}\right) + \ln(\sqrt{2}) = 6 + \ln\left(\frac{\sqrt{2}}{\frac{1}{2}\sqrt{5}}\right) = 6 + \ln\left(\frac{2}{\sqrt{5}}\sqrt{10}\right)$$

c $4\sqrt{5} - 2\sqrt{2} + \ln(4\sqrt{5}) + 6 + \ln\left(\frac{2}{\sqrt{5}}\sqrt{10}\right) + \ln(2\sqrt{2}) = 15,581...$

Dus de omtrek van W is ongeveer 15,58.

47 $G(x) = \sqrt{x^2 + 1} - \ln\left(\frac{1 + \sqrt{x^2 + 1}}{x}\right)$ geeft

$$\begin{aligned} G'(x) &= \frac{1}{2\sqrt{x^2 + 1}} \cdot 2x - \frac{1}{1 + \sqrt{x^2 + 1}} \cdot \frac{x \cdot \frac{1}{2\sqrt{x^2 + 1}} \cdot 2x - (1 + \sqrt{x^2 + 1}) \cdot 1}{x^2} \\ &= \frac{x}{\sqrt{x^2 + 1}} - \frac{x}{1 + \sqrt{x^2 + 1}} \cdot \left(\frac{1}{\sqrt{x^2 + 1}} - \frac{1 + \sqrt{x^2 + 1}}{x^2} \right) = \frac{x}{\sqrt{x^2 + 1}} - \frac{x}{1 + \sqrt{x^2 + 1}} \cdot \frac{x^2 - \sqrt{x^2 + 1} - x^2 - 1}{x^2 \sqrt{x^2 + 1}} \\ &= \frac{x}{\sqrt{x^2 + 1}} - \frac{x}{1 + \sqrt{x^2 + 1}} \cdot \frac{-1 - \sqrt{x^2 + 1}}{x^2 \sqrt{x^2 + 1}} = \frac{x}{\sqrt{x^2 + 1}} + \frac{1}{x\sqrt{x^2 + 1}} = \frac{x^2 + 1}{x\sqrt{x^2 + 1}} = \frac{\sqrt{x^2 + 1}}{x} \end{aligned}$$

Dus $G'(x) = g(x)$ oftewel G is een primitieve van g .

11.3 Inhouden

Bladzijde 118

- 48 a Het groene lichaam is te benaderen door de cilinder met straal $f(x)$ en hoogte h .

Dus de inhoud is ongeveer $\pi \cdot (f(x))^2 \cdot h$.

b $B'(x) = \lim_{h \rightarrow 0} \frac{B(x+h) - B(x)}{h} = \lim_{h \rightarrow 0} \frac{\pi \cdot (f(x))^2 \cdot h}{h} = \lim_{h \rightarrow 0} \pi \cdot (f(x))^2 = \pi \cdot (f(x))^2$

Dus $B(x)$ is een primitieve van $\pi \cdot (f(x))^2$.

Bladzijde 119

49 $f(x) = x$ geeft $2 - \sqrt{x} = x$

$$2 - x = \sqrt{x}$$

kwadrateren geeft

$$4 - 4x + x^2 = x$$

$$x^2 - 5x + 4 = 0$$

$$(x-1)(x-4) = 0$$

$$x = 1 \vee x = 4$$

vold. niet

$f(x) = 0$ geeft $2 - \sqrt{x} = 0$, oftewel $x = 4$.

$$\begin{aligned} I(K) &= \pi \int_0^1 x^2 dx + \pi \int_1^4 (2 - \sqrt{x})^2 dx = \pi \left[\frac{1}{3} x^3 \right]_0^1 + \pi \left[4x - \frac{8}{3} x\sqrt{x} + \frac{1}{2} x^2 \right]_1^4 \\ &= \frac{1}{3} \pi - 0 + \pi \left(4 \cdot 4 - \frac{8}{3} \cdot 4 \cdot 2 + \frac{1}{2} \cdot 4^2 - \left(4 \cdot 1 - \frac{8}{3} \cdot 1 \cdot 1 + \frac{1}{2} \cdot 1^2 \right) \right) = \frac{1}{3} \pi + \frac{5}{6} \pi = \frac{1}{6} \pi \end{aligned}$$

50 $g(x) = 2x$ geeft $3 - \sqrt{x} = 2x$

$$-2x + 3 = \sqrt{x}$$

kwadrateren geeft

$$4x^2 - 12x + 9 = x$$

$$4x^2 - 13x + 9 = 0$$

$$D = (-13)^2 - 4 \cdot 4 \cdot 9 = 25$$

$$x = \frac{13 \pm 5}{8} = 2\frac{1}{4} \vee x = \frac{13 - 5}{8} = 1$$

vold. niet

$g(x) = 0$ geeft $3 - \sqrt{x} = 0$, oftewel $x = 9$.

$$\begin{aligned} I(L) &= \pi \int_0^1 (2x)^2 dx + \pi \int_1^9 (3 - \sqrt{x})^2 dx = \pi \int_0^1 4x^2 dx + \pi \int_1^9 (9 - 6\sqrt{x} + x) dx \\ &= \pi \left[\frac{4}{3} x^3 \right]_0^1 + \pi \left[9x - 4x\sqrt{x} + \frac{1}{2} x^2 \right]_1^9 = \pi \left(\frac{4}{3} \cdot 1^3 - 0 \right) + \pi \left(81 - 108 + 40\frac{1}{2} - \left(9 - 4 + \frac{1}{2} \right) \right) \\ &= \frac{4}{3} \pi + 8\pi = 9\frac{1}{3} \pi \end{aligned}$$

Bladzijde 120

$$51 \quad a \quad I(K) = \pi \int_0^1 (e^x)^2 dx = \pi \int_0^1 e^{2x} dx = \pi \left[\frac{1}{2} e^{2x} \right]_0^1 = \pi \left(\frac{1}{2} e^2 - \frac{1}{2} \right)$$

$$I(L_2) = \pi \int_1^2 (e^x)^2 dx = \pi \int_1^2 e^{2x} dx = \pi \left[\frac{1}{2} e^{2x} \right]_1^2 = \pi \left(\frac{1}{2} e^4 - \frac{1}{2} e^2 \right)$$

$$\frac{I(L_2)}{I(K)} = \frac{\pi \left(\frac{1}{2} e^4 - \frac{1}{2} e^2 \right)}{\pi \left(\frac{1}{2} e^2 - \frac{1}{2} \right)} = \frac{\frac{1}{2} \pi e^2 (e^2 - 1)}{\frac{1}{2} \pi (e^2 - 1)} = e^2$$

Dus $I(L_2)$ is e^2 keer zo groot als $I(K)$.

$$b \quad I(L_p) = \pi \int_1^p (e^x)^2 dx = \pi \int_1^p e^{2x} dx = \pi \left[\frac{1}{2} e^{2x} \right]_1^p = \pi \left(\frac{1}{2} e^{2p} - \frac{1}{2} e^2 \right)$$

$$\frac{I(L_p)}{I(K)} = \frac{\pi \left(\frac{1}{2} e^{2p} - \frac{1}{2} e^2 \right)}{\pi \left(\frac{1}{2} e^2 - \frac{1}{2} \right)} = \frac{\frac{1}{2} \pi (e^{2p} - e^2)}{\frac{1}{2} \pi (e^2 - 1)} = \frac{e^{2p} - e^2}{e^2 - 1}$$

$$\frac{I(L_p)}{I(K)} = 10 \text{ geeft } \frac{e^{2p} - e^2}{e^2 - 1} = 10$$

$$e^{2p} - e^2 = 10e^2 - 10$$

$$e^{2p} = 11e^2 - 10$$

$$2p = \ln(11e^2 - 10)$$

$$p = \frac{1}{2} \ln(11e^2 - 10) = 2,1333\dots$$

Dus $p \approx 2,133$.

$$52 \quad a \quad g(x) = \frac{3}{x}$$

$$b \quad I(M) = \pi \int_1^3 \left(\frac{3}{x} \right)^2 dx = \pi \int_1^3 9x^{-2} dx = \pi \left[-9x^{-1} \right]_1^3 = \pi \left[-\frac{9}{x} \right]_1^3 = \pi \left(-\frac{9}{3} + \frac{9}{1} \right) = 6\pi$$

Wentelen van V om de lijn $y = 2$ levert hetzelfde lichaam als wentelen van W om de x -as omdat V en de lijn $y = 2$ beide 2 omlaag zijn geschoven.

$$53 \quad a \quad O(V) = \int_1^5 f(x) dx = \int_1^5 \left(3 + \frac{4}{2x-1} \right) dx = \left[3x + 4 \cdot \frac{1}{2} \ln|2x-1| \right]_1^5 = \left[3x + 2 \ln|2x-1| \right]_1^5$$

$$= 15 + 2 \ln(9) - (3 + 2 \ln(1)) = 12 + 2 \ln(9)$$

b De grafiek van f wentelen om de lijn $y = 3$ komt neer op het wentelen van de grafiek van $g(x) = f(x) - 3 = \frac{4}{2x-1}$ wentelen om de x -as.

$$I(K) = \pi \int_1^5 (g(x))^2 dx = \pi \int_1^5 \left(\frac{4}{2x-1} \right)^2 dx = \pi \int_1^5 \frac{16}{(2x-1)^2} dx = \pi \int_1^5 16(2x-1)^{-2} dx$$

$$= \pi \left[16 \cdot \frac{1}{2} \cdot -(2x-1)^{-1} \right]_1^5 = \pi \left[\frac{-8}{2x-1} \right]_1^5 = \pi \left(-\frac{8}{9} + 8 \right) = 7\frac{1}{9} \pi$$

Bladzijde 121

$$54 \quad I(L_1 + L_2) = \pi \int_2^8 (\sqrt{x-2})^2 dx = \pi \int_2^8 (x-2) dx = \pi \left[\frac{1}{2}x^2 - 2x \right]_2^8 = \pi(32 - 16) - \pi(2 - 4) = 18\pi$$

$$I(L_1) = \frac{1}{2} \cdot 18\pi \text{ geeft } \pi \int_2^a (\sqrt{x-2})^2 dx = 9\pi$$

$$\pi \int_2^a (x-2) dx = 9\pi$$

$$\int_2^a (x-2) dx = 9$$

$$\left[\frac{1}{2}x^2 - 2x \right]_2^a = 9$$

$$\left(\frac{1}{2}a^2 - 2a \right) - (2 - 4) = 9$$

$$\frac{1}{2}a^2 - 2a + 2 = 9$$

$$a^2 - 4a - 14 = 0$$

$$(a-2)^2 - 4 - 14 = 0$$

$$(a-2)^2 = 18$$

$$a-2 = \sqrt{18} \vee a-2 = -\sqrt{18}$$

$$a = 2 + 3\sqrt{2} \vee a = 2 - 3\sqrt{2}$$

vold. niet

Dus $a = 2 + 3\sqrt{2}$.

$$55 \quad a \quad O(V_p) = 2e \text{ geeft } \int_0^p e^{\frac{1}{2}x+1} dx = 2e$$

$$\left[2e^{\frac{1}{2}x+1} \right]_0^p = 2e$$

$$2e^{\frac{1}{2}p+1} - 2e^1 = 2e$$

$$2e \cdot e^{\frac{1}{2}p} = 4e$$

$$e^{\frac{1}{2}p} = 2$$

$$\frac{1}{2}p = \ln(2)$$

$$p = 2\ln(2)$$

$$b \quad I(L_p) = 9\pi e^2 \text{ geeft } \pi \int_0^p (e^{\frac{1}{2}x+1})^2 dx = 9\pi e^2$$

$$\int_0^p e^{x+2} dx = 9e^2$$

$$\left[e^{x+2} \right]_0^p = 9e^2$$

$$e^{p+2} - e^2 = 9e^2$$

$$e^p \cdot e^2 = 10e^2$$

$$e^p = 10$$

$$p = \ln(10)$$

$$56 \quad a \quad I(L) = 8\frac{2}{3}\pi - 2\frac{1}{6}\pi = 6\frac{1}{2}\pi$$

$$b \quad \pi \int_1^3 (f(x) - g(x))^2 dx = \pi \int_1^3 (x - \frac{1}{2}x)^2 dx = \pi \int_1^3 (\frac{1}{2}x)^2 dx = \pi \int_1^3 \frac{1}{4}x^2 dx =$$

$$\pi \left[\frac{1}{12}x^3 \right]_1^3 = \pi \left(\frac{1}{12} \cdot 3^3 - \frac{1}{12} \cdot 1^3 \right) = 2\frac{1}{6}\pi$$

$$\pi \int_1^3 ((f(x))^2 - (g(x))^2) dx = \pi \int_1^3 (x^2 - (\frac{1}{2}x)^2) dx = \pi \int_1^3 (x^2 - \frac{1}{4}x^2) dx = \pi \int_1^3 \frac{3}{4}x^2 dx =$$

$$\pi \left[\frac{1}{4}x^3 \right]_1^3 = \pi \left(\frac{1}{4} \cdot 3^3 - \frac{1}{4} \cdot 1^3 \right) = 6\frac{1}{2}\pi$$

Dus Irma heeft gelijk.

Bladzijde 122

57 $f(x) = g(x)$ geeft $\sqrt{x} = \frac{1}{2}x$
 kwadrateren geeft
 $x = \frac{1}{4}x^2$
 $4x = x^2$
 $x = 0 \vee x = 4$

$$I(L) = \pi \int_0^4 ((f(x))^2 - (g(x))^2) dx = \pi \int_0^4 (x - \frac{1}{4}x^2) dx = \pi \left[\frac{1}{2}x^2 - \frac{1}{12}x^3 \right]_0^4 = \pi(8 - 5\frac{1}{3}) = 2\frac{2}{3}\pi$$

Bladzijde 123

58 $f(x) = g(x)$ geeft $e^{\frac{1}{2}x} = 2e - e^{\frac{1}{2}x}$
 $2e^{\frac{1}{2}x} = 2e$
 $\frac{1}{2}x = 1$
 $x = 2$

$$I(L) = \pi \int_0^2 ((2e - e^{\frac{1}{2}x})^2 - (e^{\frac{1}{2}x})^2) dx = \pi \int_0^2 (4e^2 - 2 \cdot 2e \cdot e^{\frac{1}{2}x} + e^x - e^x) dx = \pi \int_0^2 (4e^2 - 4e^{\frac{1}{2}x+1}) dx$$

$$= \pi [4e^2x - 8e^{\frac{1}{2}x+1}]_0^2 = \pi(8e^2 - 8e^2 - (0 - 8e)) = 8\pi e$$

59 a $f(x) = 10$ geeft $\frac{10}{x\sqrt{x}} = 10$
 $x\sqrt{x} = 1$
 $x = 1$

$$O(V) = \int_1^4 \left(10 - \frac{10}{x\sqrt{x}} \right) dx = \int_1^4 (10 - 10x^{-\frac{1}{2}}) dx = [10x - 10 \cdot -2x^{\frac{1}{2}}]_1^4 = [10x + \frac{20}{\sqrt{x}}]_1^4$$

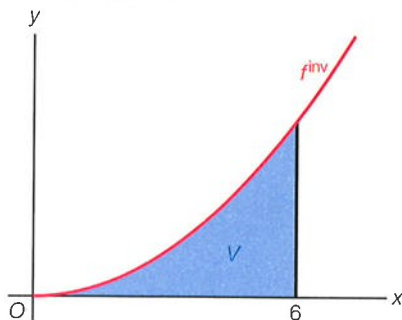
$$= 40 + 10 - (10 + 20) = 20$$

b $I(L) = \pi \int_1^4 (10^2 - (f(x))^2) dx = \pi \int_1^4 \left(100 - \frac{100}{x^3} \right) dx = \pi \int_1^4 (100 - 100x^{-3}) dx$

$$= \pi [100x + \frac{50}{x^2}]_1^4 = \pi \left(100x + \frac{50}{x^2} \right)_1^4 = \pi \left(400 + \frac{50}{16} - (100 + 50) \right) = 253\frac{1}{8}\pi$$

60 a Voor f geldt $y = 3\sqrt{x}$, dus voor f^{inv} geldt $x = 3\sqrt{y}$
 $x^2 = 9y$ met $x \geq 0$
 $y = \frac{1}{9}x^2$ met $x \geq 0$

Dus $f^{\text{inv}}(x) = \frac{1}{9}x^2$ met $x \geq 0$.



$$I(L) = \pi \int_0^6 \frac{1}{81}x^4 dx = \pi \left[\frac{1}{405}x^5 \right]_0^6 = \pi \left(\frac{1}{405} \cdot 6^5 - 0 \right) = 19\frac{1}{5}\pi$$

De gebruikte grenzen zijn 0 en 6.

b $y = 3\sqrt{x}$
 $y^2 = 9x$
 $x = \frac{1}{9}y^2$
 $x^2 = \frac{1}{81}y^4$

c Jasmijn moet de grenzen 0 en 6 gebruiken, want y loopt van 0 tot 6.

d $I(L) = \pi \int_0^6 \frac{1}{81} x^4 dx$ is hetzelfde als bij onderdeel a.

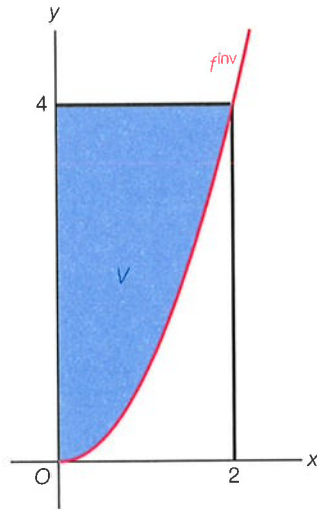
Bladzijde 125

61 a Gebruik 'buitenste' inhoud min 'binnenste' inhoud.

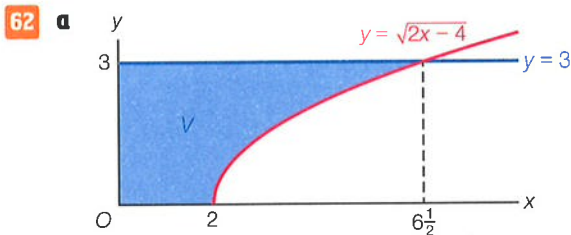
Je krijgt dan $I(L) = \pi \int_0^2 (4^2 - x^2) dy = \pi \int_0^2 (16 - y^4) dy = \pi \left[16y - \frac{1}{5} y^5 \right]_0^2 = \pi \left(32 - \frac{32}{5} \right) = 25 \frac{3}{5} \pi$.

b Voor f geldt $y = \sqrt{x}$, dus voor f^{inv} geldt $x = \sqrt{y}$
 $y = x^2$ met $x \geq 0$

Dus $f^{\text{inv}}(x) = x^2$ met $x \geq 0$.



$$I(L) = \pi \int_0^2 (4^2 - (f^{\text{inv}}(x))^2) dx = \pi \int_0^2 (16 - x^4) dx = \pi \left[16x - \frac{1}{5} x^5 \right]_0^2 = \pi \left(32 - \frac{32}{5} \right) = 25 \frac{3}{5} \pi$$



$$\begin{aligned} y = \sqrt{2x-4} \text{ geeft } y^2 &= 2x-4 \\ 2x-4 &= y^2 \\ 2x &= y^2+4 \\ x &= \frac{1}{2}y^2+2 \\ x^2 &= \left(\frac{1}{2}y^2+2\right)^2 \\ x^2 &= \frac{1}{4}y^4+2y^2+4 \end{aligned}$$

$$\begin{aligned} I(L) &= \pi \int_0^3 x^2 dy = \pi \int_0^3 \left(\frac{1}{4}y^4 + 2y^2 + 4 \right) dy = \pi \left[\frac{1}{20}y^5 + \frac{2}{3}y^3 + 4y \right]_0^3 = \pi \left(\frac{1}{20} \cdot 3^5 + \frac{2}{3} \cdot 3^3 + 4 \cdot 3 - 0 \right) \\ &= \pi \left(12 \frac{3}{20} + 18 + 12 \right) = 42 \frac{3}{20} \pi \end{aligned}$$

b $y = 3$ geeft $\sqrt{2x-4} = 3$
 $2x-4 = 9$
 $2x = 13$
 $x = 6 \frac{1}{2}$

$$\begin{aligned} I(M) &= I(\text{cilinder}) - \pi \int_2^{6\frac{1}{2}} (\sqrt{2x-4})^2 dx = \pi \cdot 3^2 \cdot 6 \frac{1}{2} - \pi \int_2^{6\frac{1}{2}} (2x-4) dx = 58 \frac{1}{2} \pi - \pi [x^2 - 4x]_2^{6\frac{1}{2}} \\ &= 58 \frac{1}{2} \pi - \pi \left((6 \frac{1}{2})^2 - 4 \cdot 6 \frac{1}{2} - (2^2 - 4 \cdot 2) \right) = 58 \frac{1}{2} \pi - 20 \frac{1}{4} \pi = 38 \frac{1}{4} \pi \end{aligned}$$

63 a $I(L_4) = \pi \int_0^4 x^2 dy = \pi \int_0^4 y dy = \pi \left[\frac{1}{2} y^2 \right]_0^4 = \pi(8 - 0) = 8\pi$

b $P(p, q)$ ligt op de parabool, dus $q = p^2$.

$$I(L_p) = I(M_q) \text{ geeft } \pi \int_0^p y^2 dx = \pi \int_0^q x^2 dy$$

$$\int_0^p x^2 dx = \int_0^{p^2} y dy$$

$$\left[\frac{1}{3} x^3 \right]_0^p = \left[\frac{1}{2} y^2 \right]_0^{p^2}$$

$$\frac{1}{3} p^3 - 0 = \frac{1}{2} p^4 - 0$$

$$2p^3 = 3p^4$$

$$p^4 = 0 \vee 2p = 3$$

$$p = 0 \vee p = \frac{3}{2}$$

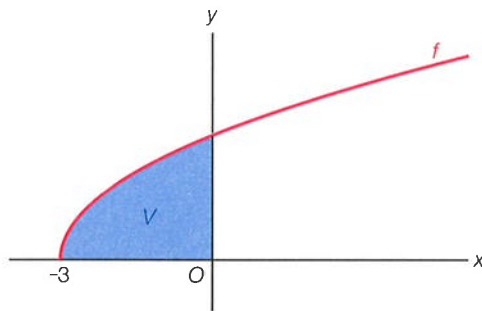
$p = 0$ voldoet niet en $p = \frac{3}{2}$ geeft $q = \frac{9}{4}$.

Dus voor $p = \frac{3}{2}$ en $q = \frac{9}{4}$.

64 a $f(x) = 0$ geeft $2x + 6 = 0$

$$2x = -6$$

$$x = -3$$



$$I(L) = \pi \int_{-3}^0 (\sqrt{2x+6})^2 dx = \pi \int_{-3}^0 (2x+6) dx = \pi \left[x^2 + 6x \right]_{-3}^0 = \pi(0 - (9 - 18)) = 9\pi$$

b $f(0) = \sqrt{6}$
 $y = \sqrt{2x+6}$ geeft $y^2 = 2x+6$

$$2x + 6 = y^2$$

$$2x = y^2 - 6$$

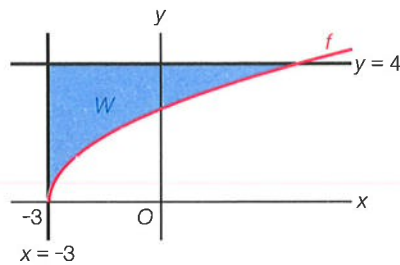
$$x = \frac{1}{2} y^2 - 3$$

$$x^2 = \left(\frac{1}{2} y^2 - 3 \right)^2$$

$$x^2 = \frac{1}{4} y^4 - 3y^2 + 9$$

$$\begin{aligned} I(M) &= \pi \int_0^{\sqrt{6}} x^2 dy = \pi \int_0^{\sqrt{6}} \left(\frac{1}{4} y^4 - 3y^2 + 9 \right) dy = \pi \left[\frac{1}{20} y^5 - y^3 + 9y \right]_0^{\sqrt{6}} \\ &= \pi \left(\frac{1}{20} \cdot (\sqrt{6})^5 - (\sqrt{6})^3 + 9 \cdot \sqrt{6} - 0 \right) = \pi \left(\frac{1}{20} \cdot 36\sqrt{6} - 6\sqrt{6} + 9\sqrt{6} \right) = 4\frac{4}{5} \pi \sqrt{6} \end{aligned}$$

c



$$y = \sqrt{2x+6}$$

↓ translatie (3, 0)

$$y = \sqrt{2(x-3)+6}$$

$$\text{oftewel } y = \sqrt{2x}$$

$$y = \sqrt{2x} \text{ geeft } y^2 = 2x$$

$$2x = y^2$$

$$x = \frac{1}{2}y^2$$

$$x^2 = \frac{1}{4}y^4$$

$$I(N) = \pi \int_0^4 x^2 dy = \pi \int_0^4 \frac{1}{4} y^4 dy = \pi \left[\frac{1}{20} y^5 \right]_0^4 = \pi \left(\frac{1}{20} \cdot 4^5 - 0 \right) = 51 \frac{1}{5} \pi$$

Bladzijde 126

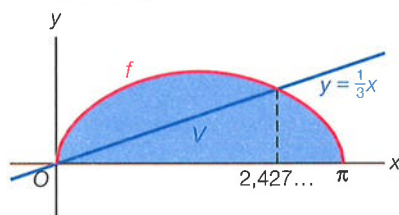
65 a $I(L) = \pi \int_{-4}^4 (x^2 + 4) dx = \pi \left[\frac{1}{3} x^3 + 4x \right]_{-4}^4 = \pi \left(\frac{64}{3} + 16 - \left(-\frac{64}{3} - 16 \right) \right) = 74 \frac{2}{3} \pi$

b $O(V) = \int_{-4}^4 \sqrt{x^2 + 4} dx$ en van $\sqrt{x^2 + 4}$ is geen primitieve te vinden met de regels die tot nu toe zijn geweest.

66 Voer in $y_1 = \sqrt{x^2 + 4}$.

De optie integraal geeft $O(V) = \int_{-4}^4 \sqrt{x^2 + 4} dx \approx 23,66$.

67 Voer in $y_1 = \sqrt{\sin(x)}$ en $y_2 = \frac{1}{3}x$.
De optie snijpunt geeft $x = 0$ en $x = 2,427...$



De optie integraal geeft $\int_0^{2,427...} (\sqrt{\sin(x)} - \frac{1}{3}x) dx = 1,019...$ en $\int_0^{\pi} \sqrt{\sin(x)} dx = 2,396...$

1,019... is niet gelijk aan de helft van 2,396..., dus de lijn $y = \frac{1}{3}x$ verdeelt het vlakdeel V niet in twee delen met gelijke oppervlakte.

Bladzijde 127

68 a $f(x) = 0$ geeft $2 \ln(x) - \ln^2(x) = 0$
 $\ln(x)(2 - \ln(x)) = 0$
 $\ln(x) = 0 \vee \ln(x) = 2$
 $x = 1 \vee x = e^2$

Dus de nulpunten zijn 1 en e^2 .

$$\text{b } f(x) = 2 \ln(x) - \ln^2(x) \text{ geeft } f'(x) = \frac{2}{x} - 2 \ln(x) \cdot \frac{1}{x} = \frac{2 - 2 \ln(x)}{x}$$

$$f'(x) = 0 \text{ geeft } 2 - 2 \ln(x) = 0$$

$$\ln(x) = 1$$

$$x = e$$

$$f(e) = 2 \cdot 1 - 1^2 = 1, \text{ dus de top is } (e, 1).$$

$$\text{c } F(x) = 4x \ln(x) - x \ln^2(x) - 4x \text{ geeft}$$

$$F'(x) = 4 \cdot \ln(x) + 4x \cdot \frac{1}{x} - \left(1 \cdot \ln^2(x) + x \cdot 2 \ln(x) \cdot \frac{1}{x} \right) - 4 = 4 \ln(x) + 4 - \ln^2(x) - 2 \ln(x) - 4$$

$$= 2 \ln(x) - \ln^2(x)$$

$$\text{Dus } F(x) = 4x \ln(x) - x \ln^2(x) - 4x \text{ is een primitieve van } f(x) = 2 \ln(x) - \ln^2(x).$$

$$\text{d } O(V) = \int_1^{e^2} (2 \ln(x) - \ln^2(x)) dx = [4x \ln(x) - x \ln^2(x) - 4x]_1^{e^2}$$

$$= 4e^2 \cdot \ln(e^2) - e^2 \cdot (\ln(e^2))^2 - 4e^2 - (4 \ln(1) - (\ln(1))^2 - 4) = 8e^2 - 4e^2 - 4e^2 - (0 - 0 - 4) = 4$$

$$\text{e } \text{De optie integraal geeft } \pi \int_1^{e^2} (2 \ln(x) - \ln^2(x))^2 dx = 9,7780...$$

$$\text{Dus } I(L) \approx 9,778.$$

11.4 Toepassingen van integralen

Bladzijde 129

$$69 \quad I_{\text{bol}} = 2 \cdot \pi \int_0^r y^2 dx = 2\pi \int_0^r (r^2 - x^2) dx = 2\pi \left[r^2 x - \frac{1}{3} x^3 \right]_0^r = 2\pi \left(r^3 - \frac{1}{3} r^3 - 0 \right) = 2\pi \cdot \frac{2}{3} r^3 = \frac{4}{3} \pi r^3$$

Bladzijde 130

$$70 \quad \text{a } I(T) = \pi \int_{\frac{1}{4}r}^{\frac{1}{2}r} (r^2 - x^2) dx = \pi \left[r^2 x - \frac{1}{3} x^3 \right]_{\frac{1}{4}r}^{\frac{1}{2}r} = \pi \left(r^2 \cdot \frac{1}{2}r - \frac{1}{3} \cdot \left(\frac{1}{2}r \right)^3 - \left(r^2 \cdot \frac{1}{4}r - \frac{1}{3} \cdot \left(\frac{1}{4}r \right)^3 \right) \right)$$

$$= \pi \left(\frac{1}{2} r^3 - \frac{1}{24} r^3 - \frac{1}{4} r^3 + \frac{1}{192} r^3 \right) = \frac{41}{192} \pi r^3$$

$$\frac{I(T)}{I(S)} = \frac{\frac{41}{192} \pi r^3}{\frac{29}{192} \pi r^3} = \frac{41}{29} = 1 \frac{12}{29}$$

Dus de inhoud van bolschijf T is $1 \frac{12}{29}$ keer zo groot als de inhoud van bolschijf S .

$$\text{b } I_p = 24\pi \text{ geeft } \frac{1}{3} \cdot \pi \cdot \left(\frac{2}{3} \cdot 6 \right)^2 \cdot 6 - \frac{1}{3} \cdot \pi \cdot \left(\frac{2}{3} p \right)^2 \cdot p = 24\pi$$

$$32 - \frac{4}{27} p^3 = 24$$

$$-\frac{4}{27} p^3 = -8$$

$$p^3 = 54$$

$$p = \sqrt[3]{54}$$

Bladzijde 131

$$71 \quad I = \pi \int_{\frac{1}{3}r}^r y^2 dx = \pi \int_{\frac{1}{3}r}^r (r^2 - x^2) dx = \pi \left[r^2 x - \frac{1}{3} x^3 \right]_{\frac{1}{3}r}^r = \pi \left(r^3 - \frac{1}{3} r^3 - \left(r^2 \cdot \frac{1}{3}r - \frac{1}{3} \cdot \left(\frac{1}{3}r \right)^3 \right) \right)$$

$$= \pi \left(r^3 - \frac{1}{3} r^3 - \frac{1}{3} r^3 + \frac{1}{81} r^3 \right) = \frac{28}{81} \pi r^3$$

$$\begin{aligned}
 72 \quad I(S_p) &= \pi \int_{-3}^p y^2 dx = \pi \int_{-3}^p (36 - x^2) dx = \pi \left[36x - \frac{1}{3}x^3 \right]_{-3}^p = \pi \left(36p - \frac{1}{3}p^3 - \left(36 \cdot -3 - \frac{1}{3} \cdot (-3)^3 \right) \right) = \pi \left(36p - \frac{1}{3}p^3 + 99 \right) \\
 I_{\text{bol}} &= \frac{4}{3} \pi r^3 = \frac{4}{3} \cdot \pi \cdot 6^3 = 288\pi \\
 I(S_p) - \frac{1}{2} \cdot 288\pi &\text{ geeft } 36p - \frac{1}{3}p^3 + 99 = 144 \\
 36p - \frac{1}{3}p^3 &= 45
 \end{aligned}$$

Voer in $y_1 = 36x - \frac{1}{3}x^3$ en $y_2 = 45$.

De optie snijpunt geeft $x = 1,268\dots$ en $x = 9,699\dots$

Dus $p \approx 1,27$.

$$\begin{aligned}
 73 \quad \text{rc}_{OA} &= \frac{4-0}{3-0} = \frac{4}{3} \\
 k \perp OA, \text{ dus } \text{rc}_k \cdot \text{rc}_{OA} &= -1 \quad \left. \begin{array}{l} \text{rc}_k \cdot \frac{4}{3} = -1 \\ \text{rc}_k = -\frac{3}{4} \end{array} \right\} \\
 \text{rc}_{OA} &= \frac{4}{3}
 \end{aligned}$$

$$\begin{aligned}
 y &= -\frac{3}{4}x + b \\
 \text{door } A(3,4) &\left. \begin{array}{l} -\frac{3}{4} \cdot 3 + b = 4 \\ -2\frac{1}{4} + b = 4 \end{array} \right\} \\
 b &= 6\frac{1}{4}
 \end{aligned}$$

Dus $k: y = -\frac{3}{4}x + 6\frac{1}{4}$.

$$\begin{aligned}
 k \text{ snijden met de } x\text{-as geeft } -\frac{3}{4}x + 6\frac{1}{4} &= 0 \\
 -\frac{3}{4}x &= -6\frac{1}{4} \\
 x &= 8\frac{1}{3}
 \end{aligned}$$

$$\begin{aligned}
 I(L) &= \pi \int_3^{8\frac{1}{3}} \left(-\frac{3}{4}x + 6\frac{1}{4} \right)^2 dx - \pi \int_3^5 (25 - x^2) dx = \pi \left[-\frac{4}{3} \cdot \frac{1}{3} \cdot \left(-\frac{3}{4}x + 6\frac{1}{4} \right)^3 \right]_3^{8\frac{1}{3}} - \pi \left[25x - \frac{1}{3}x^3 \right]_3^5 \\
 &= \pi \left[-\frac{4}{9} \left(-\frac{3}{4}x + 6\frac{1}{4} \right)^3 \right]_3^{8\frac{1}{3}} - \pi \left[25x - \frac{1}{3}x^3 \right]_3^5 \\
 &= \pi \left(-\frac{4}{9} \left(-\frac{3}{4} \cdot \frac{25}{3} + 6\frac{1}{4} \right)^3 + \frac{4}{9} \left(-\frac{3}{4} \cdot 3 + 6\frac{1}{4} \right)^3 \right) - \pi \left(25 \cdot 5 - \frac{1}{3} \cdot 5^3 - \left(25 \cdot 3 - \frac{1}{3} \cdot 3^3 \right) \right) \\
 &= \pi \left(-\frac{4}{9} \cdot 0^3 + \frac{4}{9} \cdot 4^3 \right) - \pi \left(125 - \frac{1}{3} \cdot 5^3 - \left(75 - \frac{1}{3} \cdot 3^3 \right) \right) = \pi \left(0 + \frac{4}{9} \cdot 64 \right) - \pi \left(83\frac{1}{3} - 66 \right) = 11\frac{1}{9}\pi
 \end{aligned}$$

$$74 \quad \text{a} \quad \text{De gemiddelde snelheid gedurende de eerste zes seconden is } \frac{0+10}{2} = 5 \text{ m/s.}$$

Er wordt $5 \cdot 6 = 30$ m afgelegd gedurende deze zes seconden.

$$\text{b} \quad \text{De afgelegde afstand is een primitieve van de snelheid. Dus de afstand gedurende de eerste zes seconden is } \int_0^6 v(t) dt. \text{ Ook is } O(\triangle OAB) = \int_0^6 v(t) dt.$$

Dus de oppervlakte van driehoek OAB geeft de afgelegde afstand gedurende de eerste zes seconden.

$$\text{c} \quad \text{De afgelegde afstand op het interval } [0, 15] \text{ is } O(\text{gehele driehoek}) = \frac{1}{2} \cdot 15 \cdot 10 = 75 \text{ m.}$$

Bladzijde 133

$$75 \quad \text{Stel } s(0) = c. \text{ Dan geldt } s(t) = -\frac{1}{12}t^3 + 2\frac{1}{2}t^2 + 4t + c \text{ en dit geeft } s(10) = 206\frac{2}{3} + c \text{ meter.}$$

In de eerste 10 seconden wordt dan afgelegd $s(10) - s(0) = 206\frac{2}{3} + c - c = 206\frac{2}{3}$ meter.

Je krijgt dus hetzelfde antwoord.

76 Stel $a(t) = mt + n$.

$$m = \frac{a(20) - a(0)}{20 - 0} = \frac{5 - 0}{20} = \frac{1}{4}$$

$$a(t) = \frac{1}{4}t + n \text{ en } a(0) = 0 \text{ geeft } a(t) = \frac{1}{4}t.$$

$$a(t) = \frac{1}{4}t \text{ en } v(0) = 1 \text{ geeft } v(t) = \frac{1}{8}t^2 + 1$$

$$v(t) = \frac{1}{8}t^2 + 1 \text{ en } s(0) = 0 \text{ geeft } s(t) = \frac{1}{24}t^3 + t$$

De afgelegde afstand gedurende deze 20 seconden is $s(20) = \frac{1}{24} \cdot 20^3 + 20 = 353\frac{1}{3}$ meter.

77 a $a(t) = -t^2 + 6t$ en $v(0) = 0$ geeft $v(t) = -\frac{1}{3}t^3 + 3t^2$

$$v(6) = -\frac{1}{3} \cdot 6^3 + 3 \cdot 6^2 = 36$$

Dus de snelheid op $t = 6$ is 36 m/s.

b $v(t) = -\frac{1}{3}t^3 + 3t^2$ en $s(0) = 0$ geeft $s(t) = -\frac{1}{12}t^4 + t^3$

De afgelegde afstand gedurende deze 6 seconden is $s(6) = -\frac{1}{12} \cdot 6^4 + 6^3 = 108$ meter.

c $s(6) = 108$ en $v(6) = 36$ geeft $s(10) = 108 + 4 \cdot 36 = 252$

Dus op $t = 10$ is in totaal 252 meter afgelegd.

d $500 - 108 = 392$

392 meter met een snelheid van 36 m/s duurt $\frac{392}{36} = 10\frac{8}{9}$ seconden.

Dus op $t = 6 + 10\frac{8}{9} = 16\frac{8}{9}$ is in totaal 500 meter afgelegd.

78 a 54 km/uur = 15 m/s

De constante versnelling is a .

$$v(t) = at + b \text{ en } v(0) = 15 \text{ geeft } v(t) = at + 15.$$

$$v(t) = at + 15 \text{ en } s(0) = 0 \text{ geeft } s(t) = \frac{1}{2}at^2 + 15t.$$

Noem de remtijd t_r , dan geldt $v(t_r) = 0$ en $s(t_r) = 0,75$.

$$v(t_r) = 0 \text{ geeft } at_r + 15 = 0$$

$$at_r = -15$$

$$s(t_r) = 0,75 \text{ geeft } \frac{1}{2}at_r^2 + 15t_r = 0,75 \text{ oftewel } \frac{1}{2}at_r \cdot t_r + 15t_r = 0,75.$$

$$\text{Substitutie van } at_r = -15 \text{ in } \frac{1}{2}at_r \cdot t_r + 15t_r = 0,75 \text{ geeft } -7,5t_r + 15t_r = 0,75$$

$$7,5t_r = 0,75$$

$$t_r = 0,1$$

De botsing duurt 0,1 seconde.

Alternatieve uitwerking

Het rekenwerk wordt eenvoudiger als het probleem 'achterstevoren' wordt aangepakt.

Noem de remtijd t_r en neem $v(0) = 0$, $s(0) = 0$, $v(t_r) = 15$ en $s(t_r) = 0,75$.

Dit geeft $v(t) = at$ en $s(t) = \frac{1}{2}at^2$. Dan geldt

$$\left. \begin{array}{l} at_r = 15 \\ \frac{1}{2}at_r^2 = 0,75 \text{ oftewel } \frac{1}{2}at_r \cdot t_r = 0,75 \end{array} \right\} \begin{array}{l} 7,5t_r = 0,75 \\ t_r = 0,1 \end{array}$$

De botsing duurt 0,1 seconde.

$$\left. \begin{array}{l} at_r = -15 \\ t_r = 0,1 \end{array} \right\} \begin{array}{l} a \cdot 0,1 = -15 \\ a = -150 \end{array}$$

De versnelling is -150 m/s^2 .

Dit is 15 keer zo groot als de versnelling van de zwaartekracht.

Bladzijde 134

79 a $v(t) = -0,2t^2 + 4t$ en $s(0) = 0$ geeft $s(t) = -\frac{2}{30}t^3 + 2t^2$

$$s(10) = -\frac{2}{30} \cdot 10^3 + 2 \cdot 10^2 = 133\frac{1}{3}$$

$$v(t) = 0,8t^2 - 24t + 180 \text{ geeft } s(t) = \frac{4}{15}t^3 - 12t^2 + 180t + c$$

$$s(10) = 133\frac{1}{3} \text{ geeft } \frac{4}{15} \cdot 10^3 - 12 \cdot 10^2 + 180 \cdot 10 + c = 133\frac{1}{3}$$

$$866\frac{2}{3} + c = 133\frac{1}{3}$$

$$c = -733\frac{1}{3}$$

$$\text{Dus } s(t) = \frac{4}{15}t^3 - 12t^2 + 180t - 733\frac{1}{3}.$$

$$s(15) = \frac{4}{15} \cdot 15^3 - 12 \cdot 15^2 + 180 \cdot 15 - 733\frac{1}{3} = 166\frac{2}{3}$$

De totale afstand die de auto gedurende deze 15 seconden aflegt is $166\frac{2}{3}$ meter.

b gemiddelde snelheid $= \frac{166\frac{2}{3}}{15} = 11\frac{1}{9}$ m/s

$$\text{Voer in } y_1 = -0,2x^2 + 4x, y_2 = 0,8x^2 - 24x + 180 \text{ en } y_3 = 11\frac{1}{9}.$$

De optie snijpunt met y_1 en y_3 geeft $x = 3,333...$

De optie snijpunt met y_2 en y_3 geeft $x = 11,273...$

Dus op $t \approx 3,33$ en $t \approx 11,27$ is de snelheid gelijk aan de gemiddelde snelheid.

c Voer in $y_4 = -\frac{2}{30}x^3 + 2x^2$ en $y_5 = 100$.

De optie snijpunt met y_4 en y_5 geeft $x = 8,317...$

Dus na ongeveer 8,3 seconde heeft de auto 100 meter afgelegd.

80 a $a(t) = -4e^{-0,1t}$

$$\left. \begin{array}{l} v(t) = 40e^{-0,1t} + c \\ v(3) = 32 \end{array} \right\} \begin{array}{l} 40e^{-0,3} + c = 32 \\ 29,632... + c = 32 \\ c = 2,367... \end{array}$$

$$v(t) = 40e^{-0,1t} + 2,367...$$

$$v(0) = 40 + 2,367... = 42,367...$$

De snelheid is 42,4 m/s.

b $v(t) = 40e^{-0,1t} + 2,367...$

$$\left. \begin{array}{l} s(t) = -400e^{-0,1t} + 2,367...t + c \\ s(0) = 0 \end{array} \right\} \begin{array}{l} -400 + 0 + c = 0 \\ c = 400 \end{array}$$

$$\text{Dus } s(t) = -400e^{-0,1t} + 2,367...t + 400.$$

$$\text{Voer in } y_1 = -400e^{-0,1x} + 2,367...x + 400 \text{ en } y_2 = 800.$$

De optie snijpunt geeft $x = 168,970...$

$$v(168,970...) = 2,367...$$

De snelheid waarmee de parachutiste op de grond komt is ongeveer 2,4 m/s.

Bladzijde 135

81 a $a(t) = 5e^{0,012t}$ geeft

$$\left. \begin{array}{l} v(t) = 416\frac{2}{3}e^{0,012t} + c \\ v(0) = 0 \end{array} \right\} \begin{array}{l} 416\frac{2}{3} + c = 0 \\ c = -416\frac{2}{3} \end{array}$$

$$\text{Dus } v(t) = 416\frac{2}{3}e^{0,012t} - 416\frac{2}{3}.$$

$$v(t) = 416\frac{2}{3}e^{0,012t} - 416\frac{2}{3} \text{ geeft}$$

$$\left. \begin{array}{l} s(t) = 34722\frac{2}{9}e^{0,012t} - 416\frac{2}{3}t + c \\ s(0) = 0 \end{array} \right\} \begin{array}{l} 34722\frac{2}{9} + c = 0 \\ c = -34722\frac{2}{9} \end{array}$$

$$\text{Dus } s(t) = 34722\frac{2}{9}e^{0,012t} - 416\frac{2}{3}t - 34722\frac{2}{9}.$$

$$v(170) = 416\frac{2}{3}e^{0,012 \cdot 170} - 416\frac{2}{3} = 2787,7... \text{ m/s} \approx 10036 \text{ km/uur}$$

$$s(170) = 34722\frac{2}{9}e^{0,012 \cdot 170} - 416\frac{2}{3} \cdot 170 - 34722\frac{2}{9} = 161479,4... \text{ m} \approx 161 \text{ km}$$

Dus de snelheid na 170 seconden is 10036 km/uur en de Apollo heeft dan 161 km afgelegd.

b $a(t) = 4e^{0,0045(t-170)}$ geeft

$$\left. \begin{aligned} v(t) &= 888\frac{8}{9}e^{0,0045(t-170)} + c \\ v(170) &= 2787,7... \end{aligned} \right\} \begin{aligned} 888\frac{8}{9} + c &= 2787,7... \\ c &= 1898,8... \end{aligned}$$

Dus $v(t) = 888\frac{8}{9}e^{0,0045(t-170)} + 1898,8...$

$v(t) = 888\frac{8}{9}e^{0,0045(t-170)} + 1898,8...$ geeft

$$\left. \begin{aligned} s(t) &= 197530,8...e^{0,0045(t-170)} + 1898,8...t + c \\ s(170) &= 161479,4... \end{aligned} \right\} \begin{aligned} 197530,8... + 1898,8... \cdot 170 + c &= 161479,4... \\ 520337,9... + c &= 161479,4... \\ c &= -358858,4... \end{aligned}$$

Dus $s(t) = 197530,8...e^{0,0045(t-170)} + 1898,8...t - 358858,4...$

$v(560) = 888\frac{8}{9}e^{0,0045(560-170)} + 1898,8... = 7039,7... \text{ m/s} \approx 25342 \text{ km/uur}$

$s(560) = 197530,8...e^{0,0045(560-170)} + 1898,8... \cdot 560 - 358858,4... = 1846915,3... \text{ m} \approx 1847 \text{ km}$

Dus op $t = 560$ is de snelheid 25342 km/uur en de afgelegde afstand 1847 km.

Diagnostische toets

Bladzijde 138

1 a $F(x) = x^3 e^x$ geeft $F'(x) = 3x^2 \cdot e^x + x^3 \cdot e^x = (x^3 + 3x^2)e^x$
Dus $F'(x) = f(x)$ oftewel F is een primitieve van f .

b $G(x) = \frac{1}{2}x^2 - x + 2 \ln(x+1)$ geeft $G'(x) = x - 1 + 2 \cdot \frac{1}{x+1} = \frac{(x-1)(x+1) + 2}{x+1}$
Dus $G'(x) = g(x)$ oftewel G is een primitieve van g .

2 a $f(x) = \frac{2x+6}{x^2} = 2 \cdot \frac{1}{x} + 6x^{-2}$ geeft $F(x) = 2 \ln|x| - 6x^{-1} + c = 2 \ln|x| - \frac{6}{x} + c$

b $f(x) = 3 \cdot 2^x$ geeft $F(x) = \frac{3}{\ln(2)} \cdot 2^x + c$

c $f(x) = 6e^x + x^2$ geeft $F(x) = 6e^x + \frac{1}{3}x^3 + c$

d $f(x) = \ln(x^5) = 5 \ln(x)$ geeft $F(x) = 5(x \ln(x) - x) + c = 5x \ln(x) - 5x + c$

e $f(x) = {}^2\log(4x)$ geeft $F(x) = \frac{1}{4} \cdot \frac{1}{\ln(2)} (4x \ln(4x) - 4x) + c = \frac{1}{\ln(2)} \cdot (x \ln(4x) - x) + c$

f $f(x) = 3 \sin(x) + 2 \cos(x)$ geeft $F(x) = -3 \cos(x) + 2 \sin(x) + c$

3 $f(x) = 4x - \ln(x)$ geeft $F(x) = 2x^2 - (x \ln(x) - x) + c = 2x^2 + x - x \ln(x) + c$

$$\begin{aligned} F(1) &= 7 \text{ geeft } 2 \cdot 1^2 + 1 - 1 \cdot \ln(1) + c = 7 \\ 2 + 1 - 0 + c &= 7 \\ c &= 4 \end{aligned}$$

Dus $F(x) = 2x^2 + x - x \ln(x) + 4$.

4 a $O(V) = \int_0^2 \frac{2}{x+1} dx = [2 \ln|x+1|]_0^2 = 2 \ln(3) - 2 \ln(1) = 2 \ln(3)$

b $\int_0^p \frac{2}{x+1} dx = [2 \ln|x+1|]_0^p = 2 \ln(p+1) - 2 \ln(1) = 2 \ln(p+1)$

$$\begin{aligned} \int_0^p \frac{2}{x+1} dx &= \frac{1}{2} O(V) \text{ geeft } 2 \ln(p+1) = \ln(3) \\ \ln(p+1) &= \frac{1}{2} \ln(3) \\ \ln(p+1) &= \ln(\sqrt{3}) \\ p+1 &= \sqrt{3} \\ p &= -1 + \sqrt{3} \end{aligned}$$

5 a $f(x) = (2x+6)^5 + \frac{10}{(3x-1)^2} = (2x+6)^5 + 10(3x-1)^{-2}$ geeft
 $F(x) = \frac{1}{2} \cdot \frac{1}{6} (2x+6)^6 + 10 \cdot -\frac{1}{3} (3x-1)^{-1} + c = \frac{1}{12} (2x+6)^6 - \frac{10}{3(3x-1)} + c$

b $f(x) = (5x+2)^2 \cdot \sqrt{5x+2} = (5x+2)^{\frac{5}{2}}$ geeft
 $F(x) = \frac{1}{5} \cdot \frac{2}{7} (5x+2)^{\frac{7}{2}} + c = \frac{2}{35} (5x+2)^3 \cdot \sqrt{5x+2} + c$

c $f(x) = 4e^{2x+3}$ geeft $F(x) = 4 \cdot \frac{1}{2} e^{2x+3} + c = 2e^{2x+3} + c$

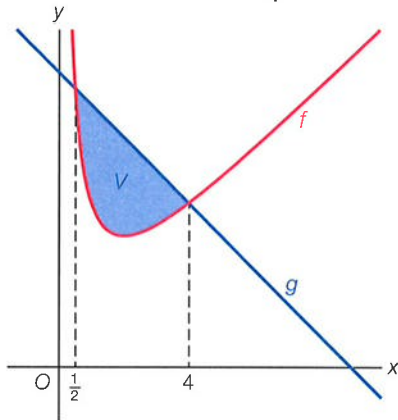
d $f(x) = 8 \cdot 2^{2x-1}$ geeft $F(x) = 8 \cdot \frac{1}{2} \cdot \frac{1}{\ln(2)} \cdot 2^{2x-1} + c = \frac{4}{\ln(2)} \cdot 2^{2x-1} + c$

e $f(x) = \ln(2x+3)$ geeft

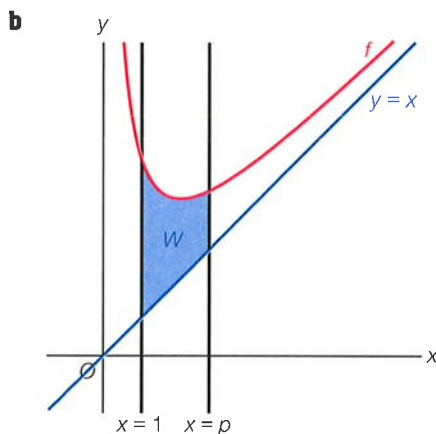
$$F(x) = \frac{1}{2} ((2x+3) \ln(2x+3) - (2x+3)) + c = (x + \frac{1}{2}) \ln(2x+3) - (x + \frac{1}{2}) + c$$

f $f(x) = \frac{6}{2x+5}$ geeft $F(x) = 6 \cdot \frac{1}{2} \cdot \ln|2x+5| + c = 3 \ln|2x+5| + c$

6 a $f(x) = g(x)$ geeft $\frac{x^2+4}{x} = -x+9$
 $x^2+4 = -x^2+9x$
 $2x^2-9x+4=0$
 $D = (-9)^2 - 4 \cdot 2 \cdot 4 = 49$
 $x = \frac{9+7}{4} = 4 \vee x = \frac{9-7}{4} = \frac{1}{2}$



$$\begin{aligned} O(V) &= \int_{\frac{1}{2}}^4 \left(-x + 9 - \frac{x^2+4}{x} \right) dx = \int_{\frac{1}{2}}^4 \left(-x + 9 - x - \frac{4}{x} \right) dx = \int_{\frac{1}{2}}^4 \left(-2x + 9 - \frac{4}{x} \right) dx \\ &= \left[-x^2 + 9x - 4 \ln|x| \right]_{\frac{1}{2}}^4 = -16 + 36 - 4 \ln(4) - \left(-\frac{1}{4} + 4\frac{1}{2} - 4 \ln\left(\frac{1}{2}\right) \right) = 20 - 4 \ln(4) - 4\frac{1}{4} + 4 \ln\left(\frac{1}{2}\right) \\ &= 15\frac{3}{4} - 8 \ln(2) - 4 \ln(2) = 15\frac{3}{4} - 12 \ln(2) \end{aligned}$$



$$O(W) = \int_1^p \left(\frac{x^2+4}{x} - x \right) dx = \int_1^p \frac{4}{x} dx = [4 \ln|x|]_1^p = 4 \ln(p) - 0 = 4 \ln(p)$$

$$\begin{aligned} O(W) &= 3 \text{ geeft } 4 \ln(p) = 3 \\ \ln(p) &= \frac{3}{4} \\ p &= e^{\frac{3}{4}} = \sqrt[4]{e^3} \end{aligned}$$

Bladzijde 139

7 $f(x) = 0$ geeft $(2x + 1)\sqrt{2x + 1} = 0$

$$(2x + 1)^{\frac{1}{2}} = 0$$

$$2x + 1 = 0$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

$$I(K) = \pi \int_{-\frac{1}{2}}^0 (2x + 1)^3 dx = \pi \left[\frac{1}{2} \cdot \frac{1}{4} (2x + 1)^4 \right]_{-\frac{1}{2}}^0 = \pi \left(\frac{1}{8} - 0 \right) = \frac{1}{8} \pi$$

$$I(L_p) = \pi \int_0^p (2x + 1)^3 dx = \pi \left[\frac{1}{2} \cdot \frac{1}{4} (2x + 1)^4 \right]_0^p = \pi \left(\frac{1}{8} (2p + 1)^4 - \frac{1}{8} \cdot 1^4 \right) = \frac{1}{8} \pi ((2p + 1)^4 - 1)$$

$$I(L_p) = 624 \cdot I(K) = 624 \cdot \frac{1}{8} \pi = 78\pi \text{ geeft } \frac{1}{8} \pi ((2p + 1)^4 - 1) = 78\pi$$

$$(2p + 1)^4 - 1 = 624$$

$$(2p + 1)^4 = 625$$

$$2p + 1 = 5 \vee 2p + 1 = -5$$

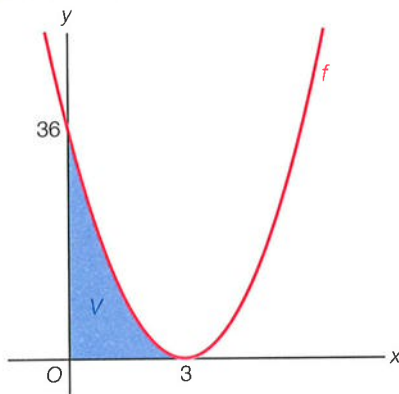
$$2p = 4 \vee 2p = -6$$

$$p = 2 \vee p = -3$$

vold. niet

Dus $p = 2$.

8 $f(0) = (-6)^2 = 36$



$$y = (2x - 6)^2 \text{ en } 0 \leq x \leq 3 \text{ geeft } 2x - 6 = -\sqrt{y}$$

$$2x = -\sqrt{y} + 6$$

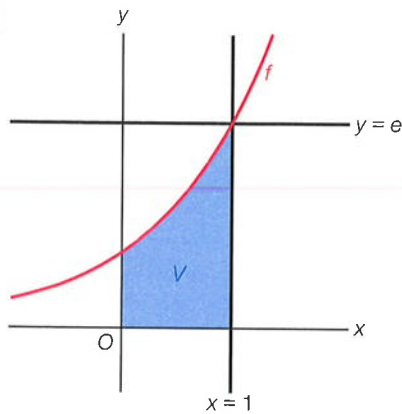
$$x = -\frac{1}{2}\sqrt{y} + 3$$

$$x^2 = \left(-\frac{1}{2}\sqrt{y} + 3\right)^2$$

$$x^2 = \frac{1}{4}y - 3\sqrt{y} + 9$$

$$\begin{aligned} I(L) &= \pi \int_0^{36} x^2 dy = \pi \int_0^{36} \left(\frac{1}{4}y - 3y^{\frac{1}{2}} + 9 \right) dy = \pi \left[\frac{1}{8}y^2 - 3 \cdot \frac{2}{3}y^{\frac{1}{2}} + 9y \right]_0^{36} = \pi \left[\frac{1}{8}y^2 - 2y\sqrt{y} + 9y \right]_0^{36} \\ &= \pi(162 - 432 + 324 - 0) = 54\pi \end{aligned}$$

9



$$f(x) = e^x$$

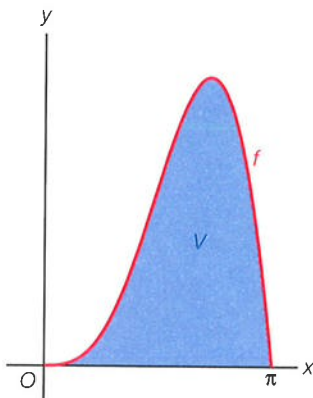
↓ translatie $(0, -e)$

$$y = e^x - e$$

Wentelen om de lijn $y = e$ geeft dezelfde inhoud als het vlakdeel ingesloten door de y -as, de lijn $x = 1$, de lijn $y = -e$ en de grafiek van $y = e^x - e$ wentelen om de x -as.

$$\begin{aligned} I(L) &= \pi \int_0^1 ((-e)^2 - (e^x - e)^2) dx = \pi \int_0^1 (e^2 - (e^{2x} - 2e \cdot e^x + e^2)) dx = \pi \int_0^1 (-e^{2x} + 2e \cdot e^x) dx \\ &= \pi \left[-\frac{1}{2} e^{2x} + 2e \cdot e^x \right]_0^1 = \pi \left(-\frac{1}{2} e^2 + 2e^2 - \left(-\frac{1}{2} + 2e \right) \right) = \left(\frac{1}{2} e^2 - 2e + \frac{1}{2} \right) \pi \approx 19,31 \end{aligned}$$

- 10 $f(x) = 0$ geeft $x^2 \sin(x) = 0$
 $x^2 = 0 \vee \sin(x) = 0$
 $x = 0 \vee x = k \cdot \pi$
 $x = 0 \vee x = \pi$



$$O(V) = \int_0^\pi x^2 \sin(x) dx$$

De optie integraal geeft $\int_0^\pi x^2 \sin(x) dx = 5,8696\dots$

Dus $O(V) \approx 5,870$.

- 11 $OA: y = 2x$

$$\begin{aligned} I(L) &= \pi \int_0^1 (2x)^2 dx + \pi \int_1^{\sqrt{5}} (5 - x^2) dx = \pi \int_0^1 4x^2 dx + \pi \int_1^{\sqrt{5}} (5 - x^2) dx = \pi \left[\frac{4}{3} x^3 \right]_0^1 + \pi \left[5x - \frac{1}{3} x^3 \right]_1^{\sqrt{5}} \\ &= \pi \left(\frac{4}{3} \cdot 1^3 - \frac{4}{3} \cdot 0^3 \right) + \pi \left(5\sqrt{5} - \frac{1}{3} (\sqrt{5})^3 - \left(5 \cdot 1 - \frac{1}{3} \cdot 1^3 \right) \right) = \frac{1}{3} \pi + \pi \left(5\sqrt{5} - \frac{1}{3} \cdot 5\sqrt{5} - 4\frac{2}{3} \right) \\ &= \pi \left(\frac{1}{3} + 3\frac{1}{3} \sqrt{5} - 4\frac{2}{3} \right) = \pi \left(3\frac{1}{3} \sqrt{5} - 3\frac{1}{3} \right) = 3\frac{1}{3} \pi (\sqrt{5} - 1) \approx 12,94 \end{aligned}$$

12 a Stel $a(t) = mt + n$ met $m = \frac{a(60) - a(0)}{60} = \frac{68 - 8}{60} = 1$ $\left. \begin{array}{l} \\ a(0) = 8 \end{array} \right\} a(t) = t + 8$

$$a(t) = t + 8 \text{ en } v(0) = 0 \text{ geeft } v(t) = \frac{1}{2}t^2 + 8t$$

$$v(t) = \frac{1}{2}t^2 + 8t \text{ en } s(0) = 0 \text{ geeft } s(t) = \frac{1}{6}t^3 + 4t^2$$

$$s(60) = \frac{1}{6} \cdot 60^3 + 4 \cdot 60^2 = 50400$$

Dus de raket bereikt in deze 60 seconden een hoogte van 50,4 km.

b Vanaf $t = 60$ is $a(t) = -10$, dus $v(t) = -10t + c$.

$$v(60) = 2280 \text{ geeft } -600 + c_1 = 2280, \text{ dus } c_1 = 2880 \text{ en } v(t) = -10t + 2880.$$

$$s(t) = -5t^2 + 2880t + c_2$$

$$s(60) = 50400 \text{ geeft } -5 \cdot 60^2 + 2880 \cdot 60 + c_2 = 50400$$

$$-18000 + 172800 + c_2 = 50400$$

$$c_2 = -104400$$

$$\text{Dus } s(t) = -5t^2 + 2880t - 104400.$$

$$s(288) = -5 \cdot 288^2 + 2880 \cdot 288 - 104400 = 310320$$

Dus de maximale hoogte is 310,32 km.